Silk Road Mathematics Competiton 2018

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1 In an acute-angled triangle ABC on the sides AB, BC, AC the points H, L, K so that $CH \perp AB$, $HL \parallel AC$, $HK \parallel BC$. Let P and Q feet of altitudes of a triangle HBL, drawn from the vertices H and B respectively. Prove that the feet of the altitudes of the triangle AKH, drawn from the vertices A and H lie on the line PQ.

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2 Find all functions $f:\ \mathbb{R} \rightarrow \mathbb{R}$ such that for any real number x the equalities are true: $f\left(x+1\right)=1+f(x)$ and $f\left(x^4-x^2\right)=f^4(x)-f^2(x).$

source (`http://matol.kz/comments/3373/show`)

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3 Given the natural n. We shall call *word* sequence from n letters of the alphabet, and *distance* $\rho(A, B)$ between *words* $A=a_1a_2\dots a_n$ and $B=b_1b_2\dots b_n$, the number of digits in which they differ (that is, the number of such i, for which $a_i\ne b_i$). We will say that the *word* C *lies* between words A and B , if $\rho (A, B)=\rho(A,C)+\rho(C,B)$. What is the largest number of *words* you can choose so that among any three, there is a *word lying* between the other two?

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4 Does there exist a sequence of positive integers $a_1, a_2, ...$ such that every positive integer occurs exactly once and that the number $\tau (na_{n+1}^n+(n+1)a_n^{n+1})$ is divisible by n for all positive integer.

Here $\tau (n)$ denotes the number of positive divisor of n.
