Silk Road Mathematics Competiton 2008

1   Suppose $a, c, d \in N$ and $d|a^2b + c$ and $d \geq a + c$
Prove that $d \geq a + \sqrt[2b]{a}$

2   In a triangle ABC A_0,B_0 and C_0 are the midpoints of the sides BC,CA and AB.A_1,B_1,C_1 are the midpoints of the broken lines BAC, CAB, ABC.Show that A_0A_1, B_0B_1, C_0C_1 are concurrent.

3   Let G be a graph with $2n$ vertexes and $2n(n-1)$ edges.If we color some edge to red,then vertexes,which are connected by this edge,must be colored to red too. But not necessary that all edges from the red vertex are red.
Prove that it is possible to color some vertexes and edges in G,such that all red vertexes has exactly n red edges.

4   Find all polynomials $P \in \mathbb{R}[x]$ such that for all $r \in \mathbb{Q}$,there exist $d \in \mathbb{Q}$ such that $P(d) = r$