Silk Road Mathematics Competiton 2017

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1 On an infinite white checkered sheet, a square Q of size 12×12 is selected. Petya wants to paint some (not necessarily all!) cells of the square with seven colors of the rainbow (each cell is just one color) so that no two of the 288 three-cell rectangles whose centers lie in Q are the same color. Will he succeed in doing this?
(Two three-celled rectangles are painted the same if one of them can be moved and possibly rotated so that each cell of it is overlaid on the cell of the second rectangle having the same color.)

(Bogdanov. I)

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2 The quadrilateral $ABCD$ is inscribed in the circle ω. The diagonals AC and BD intersect at the point O. On the segments AO and DO, the points E and F are chosen, respectively. The straight line EF intersects ω at the points E_1 and F_1. The circumscribed circles of the triangles ADE and BCF intersect the segment EF at the points E_2 and F_2 respectively (assume that all the points E, F, E_1, F_1, E_2 and F_2 are different). Prove that $E_1E_2 = F_1F_2$.

$(N.Sedrakyan)$

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3 Prove that among any 42 numbers from the interval $[1, 10^6]$, you can choose four numbers so that for any permutation (a, b, c, d) of these numbers, the inequality

$$25(ab + cd)(ad + bc) \geq 16(ac + bd)^2$$

holds.

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4 Prove that for each prime $P = 9k + 1$,exist natural n such that $P|n^3 - 3n + 1$.
