Silk Road Mathematics Competiton 2009

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1 Prove that, abc1 and a,b,c¿0

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq 1+\frac{6}{a+b+c}$$

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2 Bisectors of triangle ABC of an angles A and C intersect with BC and AB at points A1 and C1 respectively. Lines AA1 and CC1 intersect circumcircle of triangle ABC at points A2 and C2 respectively. K is intersection point of C1A2 and A1C2. I is incenter of ABC. Prove that the line KI divides AC into two equal parts.

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3 A tourist going to visit the *Complant*, found that:
a) in this country 1024 cities, numbered by integers from 0 to 1023 ,
b) two cities with numbers m and n are connected by a straight line if and only if the binary entries of numbers m and n they differ exactly in one digit,
c) during the stay of a tourist in that country 8 roads will be closed for scheduled repairs.
Prove that a tourist can make a closed route along the existing roads of *Complant*, passing through each of its cities exactly once.

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4 Prove that for any prime number p there are infinitely many fours (x,y,z,t) pairwise distinct natural numbers such that the number $(x^2+pt^2)(y^2+pt^2)(z^2+pt^2)$ is a perfect square.
