Silk Road Mathematics Competiton 2010

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1   In a convex quadrilateral it is known $ABCD$ that $\angle ADB + \angle ACB = \angle CAB + \angle DBA = 30°$ and $AD = BC$. Prove that from the lengths DB, CA and DC, you can make a right triangle.

2   Let $N = 2010! + 1$. Prove that
a) N is not divisible by 4021;
b) N is not divisible by $2027, 2029, 2039$;
c)N has a prime divisor greater than 2050.

3   For positive real numbers a, b, c, d, satisfying the following conditions: $a(c^2 - 1) = b(b^2 + c^2)$ and $d \leq 1$, prove that : $d(a\sqrt{1 - d^2} + b^2\sqrt{1 + d^2}) \leq \frac{(a+b)c}{2}$

4   In country there are two capitals (A and B) and finite number of towns.
Some towns (or town with one of capital) connected with roads (one-way). (between every two towns or capital and town there are arbitrary number of roads) such that exist at least one way from A to B.
Given, that any two ways from A to B have at least one common road.
Prove, that exist one road, such that all ways from A to B pass through this road.