Silk Road Mathematics Competiton 2011

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1 Determine the smallest possible value of $|A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5|$, where A_1, A_2, A_3, A_4, A_5 sets simultaneously satisfying the following conditions: (i) $|A_i \cap A_j| = 1$ for all $1 \leq i < j \leq 5$, i.e. any two distinct sets contain exactly one element in common; (ii) $A_i \cap A_j \cap A_k \cap A_l = \varnothing$ for all $1 \leq i < j < k < l \leq 5$, i.e. any four different sets contain no common element.
Where $|S|$ means the number of elements of S.

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2 Given an isosceles triangle ABC with base AB. Point K is taken on the extension of the side AC (beyond the point C) so that $\angle KBC = \angle ABC$. Denote S the intersection point of angle - bisectors of $\angle BKC$ and $\angle ACB$. Lines AB and KS intersect at point L, lines BS and CL intersect at point M . Prove that line KM passes through the midpoint of the segment BC.

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3 For all $a, b, c \in R^+$ such that $a + b + c = 1$ and

$$(\frac{1}{(a+b)^2} + \frac{1}{(b+c)^2} + \frac{1}{(c+a)^2})(a-bc)(b-ac)(c-ab) \leq M \cdot abc.$$

Find min M.

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4 Prove that there are infinitely many primes representable in the form $m^2 + mn + n^2$ for some integers m, n .
