

Silk Road Mathematics Competiton 2012

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- 1 Trapezium $ABCD$, where $BC \parallel AD$, is inscribed in a circle, E is midpoint of the arc AD of this circle not containing point C . Let F be the foot of the perpendicular drawn from E on the line tangent to the circle at the point C . Prove that $BC = 2CF$.

- 2 In each cell of the table 4×4 , in which the lines are labeled with numbers 1, 2, 3, 4, and columns with letters a, b, c, d , one number is written: 0 or 1. Such a table is called *valid* if there are exactly two units in each of its rows and in each column. Determine the number of *valid* tables.

- 3 Let $n > 1$ be an integer.
Determine the greatest common divisor of the set of numbers $\left\{ \binom{2n}{2i+1} : 0 \leq i \leq n-1 \right\}$
i.e. the largest positive integer, dividing $\binom{2n}{2i+1}$ without remainder for every $i = 0, 1, \dots, n-1$.
(Here $\binom{m}{l} = C_m^l = \frac{m!}{l!(m-l)!}$ is binomial coefficient.)

- 4 Prove that for any positive integer n , the arithmetic mean of $\sqrt[1]{1}, \sqrt[2]{2}, \sqrt[3]{3}, \dots, \sqrt[n]{n}$ lies in $\left[1, 1 + \frac{2\sqrt{2}}{\sqrt{n}} \right]$.