Silk Road Mathematics Competiton 2013

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1 Determine all pairs of positive integers $m, n,$ satisfying the equality $(2^m+1;2^n+1)=2^{(m;n)}+1$, where $(a;b)$ is the greatest common divisor

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2 Circle with center I, inscribed in a triangle ABC , touches the sides BC and AC at points A_1 and B_1 respectively. On rays A_1I and B_1I, respectively, let be the points A_2 and B_2 such that $IA_2 = IB_2 = R$, where $R$is the radius of the circumscribed circle of the triangle ABC. Prove that:
a) $AA_2 = BB_2 = OI$ where O is the center of the circumscribed circle of the triangle ABC,
b) lines AA_2 and BB_2 intersect on the circumcircle of the triangle ABC.

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3 Find all non-decreasing functions $f : \mathbb{N} \to \mathbb{N}$, such that $f(f(m)f(n)+m) = f(mf(n)) + f(m)$

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4 In the film there is n roles. For each i ($1 \le i \le n$), the role of number i can play a_i a person, and one person can play only one role. Every day a casting is held, in which participate people for n roles, and from each role only one person. Let p be a prime number such that $p \ge a_1, \dots, a_n, n$. Prove that you can have p^k castings such that if we take any k people who are tried in different roles, they together participated in some casting (k is a natural number not exceeding n).
