Silk Road Mathematics Competiton 2014

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1	What is the maximum number of coins can be arranged in cells of the table $n \times n$ (each cell is not more of the one coin) so that any coin was not simultaneously below and to the right than any other?
2	Let w be the circumcircle of non-isosceles acute triangle ABC. Tangent lines to w in A and B intersect at point S. Let M be the midpoint of AB, and H be the orthocenter of triangle ABC. The line HA intersects lines CM and CS at points M_a and S_a, respectively. The points M_b and S_b are defined analogously. Prove that M_aS_b and M_bS_a are the altitudes of triangle M_aM_bH.
3	$a, b, c \geq 0, \quad a^3 + b^3 + c^3 + abc = 4$ Prove that $a^3b + b^3c + c^3b \leq 3$
4	Find all $f : N \to N$, such that $\forall m, n \in N \; 2f(mn) \geq f(m^2 + n^2) - f(m)^2 - f(n)^2 \geq 2f(m)f(n)$