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- 1 Let $ABCD A' B' C' D'$ be a rectangular parallelepiped, where $ABCD$ is the lower face and A, B, C and D are below A', B', C' and D' , respectively. The parallelepiped is divided into eight parts by three planes parallel to its faces. For each vertex P , let V_P denote the volume of the part containing P . Given that $V_A = 40, V_C = 300, V_B = 360$ and $V_C = 90$, find the volume of $ABCD A' B' C' D'$.

- 2 A music streaming service proposes songs classified in 10 musical genres, so that each song belong to one and only one gender. The songs are played one after the other: the first 17 are chosen by the user, but starting from the eighteenth the service automatically determines which song to play. Elisabetta has noticed that, if one makes the classification of which genres they appear several times during the last 17 songs played, the new song always belongs to the genre at the top of the ranking or, in case of same merit, at one of the first genres. Prove that, however, the first 17 tracks are chosen, from a certain point onwards the songs proposed are all of the same kind.

- 3 Let ABC a triangle, let K be the foot of the bisector relative to BC and J be the foot of the trisectrix relative to BC closer to the side AC ($3\angle JAC = \angle CAB$). Let C' and B' be two point on the line AJ on the side of J with respect to A , such that $AC' = AC$ and $AB = AB'$. Prove that $ABB'C$ is cyclic if and only if lines $C'K$ and BB' are parallel.

- 4 Determine all pairs of integers (a, b) that solve the equation $a^3 + b^3 + 3ab = 1$.

- 5 Let AB be a chord of a circle Γ and let C be a point on the segment AB . Let r be a line through C which intersects Γ at the points D, E ; suppose that D, E lie on different sides with respect to the perpendicular bisector of AB . Let Γ_D be the circumference which is externally tangent to Γ at D and touches the line AB at F . Let Γ_E be the circumference which is externally tangent to Γ at E and touches the line AB at G . Prove that $CA = CB$ if and only if $CF = CG$.

- 6 Ada and Charles play the following game: at the beginning, an integer $n \geq 1$ is written on the blackboard. In turn, Ada and Charles remove the number k that they find on the blackboard. In turn Ada and Charles remove the number k that they find on the blackboard and they replace it :
 1- either with a positive divisor k different from 1 and k
 2- or with $k+1$
 At the beginning each players have a thousand points each. When a player choses move 1,

he/she gains one point; when a player chooses move 2, he/she loses one point. The game ends when one of the two players is left with zero points and this player loses the game. Ada moves first. For what values of n does Chara have a winning strategy?
