Spain Mathematical Olympiad 2015

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–	Day 1
1	On the graph of a polynomial with integer coefficients, two points are chosen with integer coordinates. Prove that if the distance between them is an integer, then the segment that connects them is parallel to the horizontal axis.
2	In triangle ABC, let A' is the symmetrical of A with respect to the circumcenter O of ABC. Prove that: **a)** The sum of the squares of the tangents segments drawn from A and A' to the incircle of ABC equals $$4R^2-4Rr-2r^2$$ where R and r are the radii of the circumscribed and inscribed circles of ABC respectively. **b)** The circle with center A' and radius $A'I$ intersects the circumcircle of ABC in a point L such that $$AL=\sqrt{AB.AC}$$ where I is the centre of the inscribed circle of ABC.
3	On the board is written an integer $N\geq 2$. Two players A and B play in turn, starting with A. Each player in turn replaces the existing number by the result of performing one of two operations: subtract 1 and divide by 2, provided that a positive integer is obtained. The player who reaches the number 1 wins. Determine the smallest even number N requires you to play at least 2015 times to win (B shifts are not counted).
–	Day 2
1	All faces of a polyhedron are triangles. Each of the vertices of this polyhedron is assigned independently one of three colors : green, white or black. We say that a face is *Extremadura* if its three vertices are of different colors, one green, one white and one black. Is it true that regardless of how the vertices's color, the number of *Extremadura* faces of this polyhedron is always even?
2	Let p and n be a natural numbers such that p is a prime and $1+np$ is a perfect square. Prove that the number $n+1$ is sum of p perfect squares.

3 Let ABC be a triangle. M, and N points on BC, such that $BM = CN$, with M in the interior of BN. Let P and Q be points in AN and AM respectively such that $\angle PMC = \angle MAB$, and $\angle QNB = \angle NAC$. Prove that $\angle QBC = \angle PCB$.