Rioplatense Mathematical Olympiad, Level 3 2002

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–	Day 1
1	Determine all pairs (a, b) of positive integers for which $\frac{a^2b+b}{ab^2+9}$ is an integer number.
2	Let λ be a real number such that the inequality $0 < \sqrt{2002} - \frac{a}{b} < \frac{\lambda}{ab}$ holds for an infinite number of pairs (a, b) of positive integers. Prove that $\lambda \geq 5$.
3	Let ABC be a triangle with $\angle C = 60^o$. The point P is the symmetric of A with respect to the point of tangency of the circle inscribed with the side BC . Show that if the perpendicular bisector of the CP segment intersects the line containing the angle - bisector of $\angle B$ at the point Q, then the triangle CPQ is equilateral.
–	Day 2
4	Let a, b and c be positive real numbers. Show that $\frac{a+b}{c^2} + \frac{c+a}{b^2} + \frac{b+c}{a^2} \geq \frac{9}{a+b+c} + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$
5	ABC is any triangle. Tangent at C to circumcircle (O) of ABC meets AB at M. Line perpendicular to OM at M intersects BC at P and AC at Q. P.T. $MP = MQ$.
6	Daniel chooses a positive integer n and tells Ana. With this information, Ana chooses a positive integer k and tells Daniel. Daniel draws n circles on a piece of paper and chooses k different points on the condition that each of them belongs to one of the circles he drew. Then he deletes the circles, and only the k points marked are visible. From these points, Ana must reconstruct at least one of the circumferences that Daniel drew. Determine which is the lowest value of k that allows Ana to achieve her goal regardless of how Daniel chose the n circumferences and the k points.