Rioplatense Mathematical Olympiad, Level 3 2000

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–	Day 1
1	Let a and b be positive integers such that the number $b^2+(b+1)^2+...+(b+a)^2-3$ is multiple of 5 and $a+b$ is odd. Calculate the digit of the units of the number $a+b$ written in decimal notation.
2	In a triangle ABC, points D, E and F are considered on the sides BC, CA and AB respectively, such that the areas of the triangles AFE, BFD and CDE are equal. Prove that $$\frac{(DEF)}{(ABC)} \ge \frac{1}{4}$$ Note: (XYZ) is the area of triangle XYZ.
3	Let $n > 1$ be an integer. For each numbers $(x_1, x_2, \dots, x_n)$ with $x_1^2+x_2^2+x_3^2+\dots+x_n^2=1$, denote $m=\min\{\|x_i-x_j\|, 0<i<j<n+1\}$ Find the maximum value of m.
–	Day 2
4	Let a, b and c be positive integers such that $a^2+b^2+1=c^2$. Prove that $[a/2]+[c/2]$ is even. Note: $[x]$ is the integer part of x.
5	Let ABC be a triangle with $AB<AC$, let L be midpoint of arc BC(the point A is not in this arc) of the circumcircle $w(ABC)$. Let E be a point in AC where $AE=\frac{AB+AC}{2}$, the line EL intersects w in P. If M and N are the midpoints of AB and BC, respectively, prove that AL, BP and MN are concurrents
6	Let $g(x)=ax^2+bx+c$ a quadratic function with real coefficients such that the equation $g(g(x))=x$ has four distinct real roots. Prove that there isn't a function f: $R--R$ such that $f(f(x))=g(x)$ for all x real