Puerto Rico Team Selection Test 2013

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by fprosk

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1   Claudia and Adela are betting to see which one of them will ask the boy they like for his telephone number. To decide they roll dice. If none of the dice are a multiple of 3, Claudia will do it. If exactly one die is a multiple of 3, Adela will do it. If 2 or more of the dice are a multiple of 3 neither one of them will do it. How many dice should be rolled so that the risk is the same for both Claudia and Adela?

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2   How many 3-digit numbers have the property that the sum of their digits is even?

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3   Find all pairs of natural numbers n and prime numbers p such that $\sqrt{n + \frac{p}{n}}$ is a natural number.

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4   If $x_0 = x_1 = 1$, and for $n \geq 1$

$x_{n+1} = \frac{x_n^2}{x_{n-1} + 2x_n}$,

find a formula for x_n as a function of n.

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5   Given an equilateral triangle we select an arbitrary point on its interior. We draw the perpendiculars from that point to the three sides of the triangle. Show that the sum of the lengths of these perpendiculars is equal to the height of the triangle.

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6   A 9×9 checkerboard is colored with 2 colors. If we choose any 3×1 region on the checkerboard we can paint all of the squares in that region with the color that is in the majority in that region. Show that with a finite number of these operations, we can paint the checkerboard all in the same color.

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7   Show that if $\sqrt{x} - \sqrt{y} = 10$, then $x - 2y \leq 200$.