National Science Olympiad 2010

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by parmenides51, wangsacl

– Day 1

1 Let a, b, c be three different positive integers. Show that the sequence

$$a+b+c, ab+bc+ca, 3abc$$

could be neither an arithmetic nor geometric progression.

Fajar Yuliawan, Bandung

2 Given an acute triangle ABC with $AC > BC$ and the circumcenter of triangle ABC is O. The altitude of triangle ABC from C intersects AB and the circumcircle at D and E, respectively. A line which passed through O which is parallel to AB intersects AC at F. Show that the line CO, the line which passed through F and perpendicular to AC, and the line which passed through E and parallel with DO are concurrent.

Fajar Yuliawan, Bandung

3 A mathematical competition was attended by 120 participants from several contingents. At the closing ceremony, each participant gave 1 souvenir each to every other participants from the same contingent, and 1 souvenir to any person from every other contingents. It is known that there are 3840 souvenirs whom were exchanged.
Find the maximum possible contingents such that the above condition still holds?

Raymond Christopher Sitorus, Singapore

4 Given that m and n are positive integers with property:

$$(mn) \mid (m^{2010} + n^{2010} + n)$$

Show that there exists a positive integer k such that $n = k^{2010}$

Nanang Susyanto, Yogyakarta

– Day 2

5 m boys and n girls ($m > n$) sat across a round table, supervised by a teacher, and they did a game, which went like this. At first, the teacher pointed a boy to start the game. The chosen boy put a coin on the table. Then, consecutively in a clockwise order, everyone did his turn. If the next person is a boy, he will put a coin to the existing pile of coins. If the next person is a girl, she will take a coin from the existing pile of coins. If there is no coin on the table, the game ends. Notice that depending on the chosen boy, the game could end early, or it could go for a full turn. If the teacher wants the game to go for at least a full turn, how many possible boys could be chosen?

Hendrata Dharmawan, Boston, USA

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6 Find all positive integers $n > 1$ such that

$$\tau(n) + \phi(n) = n + 1$$

Which in this case, $\tau(n)$ represents the amount of positive divisors of n, and $\phi(n)$ represents the amount of positive integers which are less than n and relatively prime with n.

Raja Oktovin, Pekanbaru

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7 Given 2 positive reals a and b. There exists 2 polynomials $F(x) = x^2 + ax + b$ and $G(x) = x^2 + bx + a$ such that all roots of polynomials $F(G(x))$ and $G(F(x))$ are real. Show that a and b are more than 6.

Raja Oktovin, Pekanbaru

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8 Given an acute triangle ABC with circumcenter O and orthocenter H. Let K be a point inside ABC which is not O nor H. Point L and M are located outside the triangle ABC such that $AKCL$ and $AKBM$ are parallelogram. At last, let BL and CM intersects at N, and let J be the midpoint of HK. Show that $KONJ$ is also a parallelogram.

Raja Oktovin, Pekanbaru
