2015 Azerbaijan JBMO TST

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–	Day 1
1	With the conditions $a, b, c \in \mathbb{R}^+$ and $a+b+c=1$, prove that $$\frac{7+2b}{1+a}+\frac{7+2c}{1+b}+\frac{7+2a}{1+c} \geq \frac{69}{4}$$
3	Acute-angled $\triangle ABC$ triangle with condition $AB<AC<BC$ has cimcumcircle C' with center O and radius R.And BD and CE diametrs drawn.Circle with center O and radius R intersects AC at K.And circle with center A and radius AD intersects BA at L.Prove that EK and DL lines intersects at circle C'.
2	$A=1\cdot 4\cdot 7\cdots 2014$.Find the last non-zero digit of A if it is known that $A \equiv 1 \mod 3$.
4	Prove that there are not intgers a and b with conditions, i) $16a-9b$ is a prime number. ii) ab is a perfect square. iii) $a+b$ is also perfect square.
–	Day 2
1	Let x, y and z be non-negative real numbers satisfying the equation $x+y+z=xyz$. Prove that $2(x^2+y^2+z^2) \geq 3(x+y+z)$.
2	All letters in the word $VUQAR$ are different and chosen from the set $\{1,2,3,4,5\}$. Find all solutions to the equation $$\frac{(V+U+Q+A+R)^2}{V-U-Q+A+R}=V^{{{U^Q}^A}^R}.$$
3	Let ABC be a triangle such that AB is not equal to AC. Let M be the midpoint of BC and H be the orthocenter of triangle ABC. Let D be the midpoint of AH and O the circumcentre of triangle BCH. Prove that $DAMO$ is a parallelogram.
–	Day 3

1 $a,b,c \in \mathbb{R}^+$ and $a^2+b^2+c^2=48$. Prove that

$$a^2\sqrt{2b^3+16}+b^2\sqrt{2c^3+16}+c^2\sqrt{2a^3+16}\le 24^2$$

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2 There are some real numbers on the board (at least two). In every step we choose two of them, for example a and b, and then we replace them with $\frac{ab}{a+b}$. We continue until there is one number. Prove that the last number does not depend on which order we choose the numbers to erase.

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3 There is a triangle ABC that AB is not equal to $AC.BD$ is interior bisector of $\angle ABC(D\in AC)$ M is midpoint of CBA arc.Circumcircle of $\triangle BDM$ cuts AB at K and J, is symmetry of A according K.If $DJ\cap AM=(O)$, Prove that J,B,M,O are cyclic.

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4 Find all integer solutions to the equation $x^2=y^2(x+y^4+2y^2)$.

– Additional Exam

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1 Let a,b,c be positive real numbers. Prove that

$$\left((3a^2+1)^2+2\left(1+\frac{3}{b}\right)^2\right)\left((3b^2+1)^2+2\left(1+\frac{3}{c}\right)^2\right)\left((3c^2+1)^2+2\left(1+\frac{3}{a}\right)^2\right)\ge 48^3$$

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2 Find all non-negative solutions to the equation $2013^x+2014^y=2015^z$
