Lusophon Mathematical Olympiad 2018

www.artofproblemsolving.com/community/c726875

by parmenides51

–	Day 1
1	Fill in the corners of the square, so that the sum of the numbers in each one of the 5 lines of the square is the same and the sum of the four corners is 123.
2	In a triangle ABC, right in A and isosceles, let D be a point on the side AC ($A \neq D \neq C$) and E be the point on the extension of BA such that the triangle ADE is isosceles. Let P be the midpoint of segment BD, R be the midpoint of the segment CE and Q the intersection point of ED and BC. Prove that the quadrilateral $ARQP$ is a square
3	For each positive integer n, let $S(n)$ be the sum of the digits of n. Determines the smallest positive integer a such that there are infinite positive integers n for which you have $S(n)-S(n+a)=2018$.
–	Day 2
4	Determine the pairs of positive integer numbers m and n that satisfy the equation $m^2=n^2+m+n+2018$.
5	Determine the increasing geometric progressions, with three integer terms, such that the sum of these terms is 57
6	In a 3×25 board, 1×3 pieces are placed (vertically or horizontally) so that they occupy entirely 3 boxes on the board and do not have a common point. What is the maximum number of pieces that can be placed, and for that number, how many configurations are there? Num tabuleiro 3 × 25 s˜ao colocadas pe¸cas 1 × 3 (na vertical ou na horizontal) de modo que ocupem inteiramente 3 casas do tabuleiro e n˜ao se toquem em nenhum ponto. Qual ´e o n´umero m´aximo de pe¸cas que podem ser colocadas, e para esse n´umero, quantas configura¸c˜oes existem? source (`https://www.obm.org.br/content/uploads/2018/09/Provas_OMCPLP_2018.pdf`)