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–	Paper 1
1	If the three-digit number ABC is divisible by 27, prove that the three-digit numbers BCA and CAB are also divisible by 27.
2	In triangle ABC we have $\|AB\| \neq \|AC\|$. The bisectors of $\angle ABC$ and $\angle ACB$ meet AC and AB at E and F, respectively, and intersect at I. If $\|EI\| = \|FI\|$ find the measure of $\angle BAC$.
3	Do there exist four polynomials $P_1(x), P_2(x), P_3(x), P_4(x)$ with real coefficients, such that the sum of any three of them always has a real root, but the sum of any two of them has no real root?
4	Let ABC be a triangle with $\|AC\| \neq \|BC\|$. Let P and Q be the intersection points of the line AB with the internal and external angle bisectors at C, so that P is between A and B. Prove that if M is any point on the circle with diameter PQ, then $\angle AMP = \angle BMP$.
5	Let $a_1, a_2, ..., a_m$ be positive integers, none of which is equal to 10, such that $a_1 + a_2 + ... + a_m = 10m$. Prove that $(a_1a_2a_3 \cdot ... \cdot a_m)^{1/m} \leq 3\sqrt{11}$.
–	Paper 2
6	Triangle ABC has sides $a = \|BC\| > b = \|AC\|$. The points K and H on the segment BC satisfy $\|CH\| = (a + b)/3$ and $\|CK\| = (a - b)/3$. If G is the centroid of triangle ABC, prove that $\angle KGH = 90^o$.
7	A rectangular array of positive integers has 4 rows. The sum of the entries in each column is 20. Within each row, all entries are distinct. What is the maximum possible number of columns?
8	Suppose a, b, c are real numbers such that $abc \neq 0$. Determine x, y, z in terms of a, b, c such that $bz + cy = a, cx + az = b, ay + bx = c$. Prove also that $\frac{1-x^2}{a^2} = \frac{1-y^2}{b^2} = \frac{1-z^2}{c^2}$.
9	Show that the number a^3 where $a = \frac{251}{\frac{1}{\sqrt[3]{252}-5\sqrt[3]{2}}-10\sqrt[3]{63}} + \frac{1}{\frac{251}{\sqrt[3]{252}+5\sqrt[3]{2}}+10\sqrt[3]{63}}$ is an integer and find its value
10	Let AE be a diameter of the circumcircle of triangle ABC. Join E to the orthocentre, H, of $\triangle ABC$ and extend EH to meet the circle again at D. Prove that the nine point circle of $\triangle ABC$

passes through the midpoint of HD.

Note. The nine point circle of a triangle is a circle that passes through the midpoints of the sides, the feet of the altitudes and the midpoints of the line segments that join the orthocentre to the vertices.