Bosnia and Herzegovina Team Selection Test 2004

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–	Day 1
1	Circle k with center O is touched from inside by two circles in points S and T, respectively. Let those two circles intersect at points M and N, such that N is closer to line ST. Prove that OM and MN are perpendicular iff S, N and T are collinear
2	Determine whether does exists a triangle with area 2004 with his sides positive integers.
3	Let a, b and c be positive real numbers such that $abc = 1$. Prove the inequality: $\frac{ab}{a^5+b^5+ab} + \frac{bc}{b^5+c^5+bc} + \frac{ac}{c^5+a^5+ac} \leq 1$
–	Day 2
4	On competition which has 16 teams, it is played 55 games. Prove that among them exists 3 teams such that they have not played any matches between themselves.
5	For $0 \leq x < \frac{\pi}{2}$ prove the inequality: $a^2 \tan(x) \cdot (\cos(x))^{\frac{1}{3}} + b^2 \sin x \geq 2xab$ where a and b are real numbers.
6	It is given triangle ABC and parallelogram $ASCR$ with diagonal AC. Let line constructed through point B parallel with CS intersects line AS and CR in M and P, respectively. Let line constructed through point B parallel with AS intersects line AR and CS in N and Q, respectively. Prove that lines RS, MN and PQ are concurrent