Bosnia and Herzegovina Team Selection Test 2003

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by gobathegreat

–	Day 1
1	Board has written numbers: 5, 7 and 9. In every step we do the following: for every pair (a,b), $a>b$ numbers from the board, we also write the number $5a-4b$. Is it possible that after some iterations, 2003 occurs at the board ?
2	Upon sides AB and BC of triangle ABC are constructed squares ABB_1A_1 and BCC_1B_2. Prove that lines AC_1, CA_1 and altitude from B to side AC are concurrent.
3	Prove that for every positive integer n holds: $(n-1)^n+2n^n \leq (n+1)^n \leq 2(n-1)^n+2n^n$
–	Day 2
4	In triangle ABC AD and BE are altitudes. Let L be a point on ED such that ED is orthogonal to BL. If $LB^2=LD \cdot LE$ prove that triangle ABC is isosceles
5	It is given regular polygon with $2n$ sides and center S. Consider every quadrilateral with vertices as vertices of polygon. Let u be number of such quadrilaterals which contain point S inside and v number of remaining quadrilaterals. Find $u-v$
6	Let a, b and c be real numbers such that $\mid a \mid > 2$ and $a^2+b^2+c^2=abc+4$. Prove that numbers x and y exist such that $a=x+\frac{1}{x}$, $b=y+\frac{1}{y}$ and $c=xy+\frac{1}{xy}$.