Regional Olympiad - Federation of Bosnia and Herzegovina 2017

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by gobathegreat

–	Bugojno, April 1st
–	Grade 9
1	Let a, b and c be real numbers such that $abc(a+b)(b+c)(c+a) \neq 0$ and $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{1007}{1008}$ Prove that $\frac{ab}{(a+c)(b+c)}+\frac{bc}{(b+a)(c+a)}+\frac{ca}{(c+b)(a+b)}=2017$
2	Prove that numbers $1, 2, ..., 16$ can be divided in sequence such that sum of any two neighboring numbers is perfect square
3	Does there exist positive integer n such that sum of all digits of number $n(4n+1)$ is equal to 2017
4	It is given isosceles triangle ABC ($AB = AC$) such that $\angle BAC = 108^{\circ}$. Angle bisector of angle $\angle ABC$ intersects side AC in point D, and point E is on side BC such that $BE = AE$. If $AE = m$, find ED
–	Grade 10
1	If a is real number such that x_1 and x_2, $x_1 \neq x_2$, are real numbers and roots of equation $x_2 - x + a = 0$. Prove that $\mid {x_1}^2 - {x_2}^2 \mid = 1$ iff $\mid {x_1}^3 - {x_2}^3 \mid = 1$
2	It is given triangle ABC. Let internal and external angle bisector of angle $\angle BAC$ intersect line BC in points D and E, respectively, and circumcircle of triangle ADE intersects line AC in point F. Prove that FD is angle bisector of $\angle BFC$
3	Find prime numbers p, q, r and s, pairwise distinct, such that their sum is prime number and numbers $p^2 + qr$ and $p^2 + qs$ are perfect squares
4	Let S be a set of n distinct real numbers, and A_S set of arithemtic means of two distinct numbers from S. For given $n \geq 2$ find minimal number of elements in A_S
–	Grade 11
1	In terms of real parameter a solve inequality: $\log_a x + \mid a + \log_a x \mid \cdot \log_{\sqrt{x}} a \geq a \log_x a$ in set of real numbers

2 Let ABC be an isosceles triangle such that $AB=AC$. Find angles of triangle ABC if $\frac{AB}{BC}=1+2\cos{\frac{2\pi}{7}}$

3 Let S be a set of 6 positive real numbers such that $(a,b \in S)$ $(a>b) \Rightarrow a+b \in S$ or $a-b \in S$ Prove that if we sort these numbers in ascending order, then they form an arithmetic progression

4 It is given positive integer N. Let d_1, d_2,...,d_n be its divisors and let a_i be number of divisors of d_i, $i=1,2,...n$. Prove that

$$(a_1+a_2+...+a_n)^2={a_1}^3+{a_2}^3+...+{a_n}^3$$

– Grade 12

1 Problem 1 for grade 11

2 In triangle ABC on side AC are points K, L and M such that BK is an angle bisector of $\angle ABL$, BL is an angle bisector of $\angle KBM$ and BM is an angle bisector of $\angle LBC$, respectively. Prove that $4 \cdot LM < AC$ and $3 \cdot \angle BAC - \angle ACB < 180^{\circ}$

3 Problem 4 for grade 11

4 How many knights you can put on chess table 5×5 such that every one of them attacks exactly two other knights ?