**Bosnia and Herzegovina European Girls Mathematical Olympiad TST 2017**

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**1** It is given sequence wih length of $2017$ which consists of first $2017$ positive integers in arbitrary order (every number occus exactly once). Let us consider a first term from sequence, let it be $k$. From given sequence we form a new sequence of length 2017, such that first $k$ elements of new sequence are same as first $k$ elements of original sequence, but in reverse order while other elements stay unchanged. Prove that if we continue transforming a sequence, eventually we will have sequence with first element $1$.

**2** It is given triangle $ABC$ and points $P$ and $Q$ on sides $AB$ and $AC$, respectively, such that $PQ \mid\mid BC$. Let $X$ and $Y$ be intersection points of lines $BQ$ and $CP$ with circumcircle $k$ of triangle $APQ$, and $D$ and $E$ intersection points of lines $AX$ and $AY$ with side $BC$. If $2 \cdot DE = BC$, prove that circle $k$ contains intersection point of angle bisector of $\angle BAC$ with $BC$

**3** For positive integer $n$ we define $f(n)$ as sum of all of its positive integer divisors (including $1$ and $n$). Find all positive integers $c$ such that there exists strictly increasing infinite sequence of positive integers $n_1, n_2, n_3, ...$ such that for all $i \in \mathbb{N}$ holds $f(n_i) - n_i = c$

**4** Let $a$, $b$, $c$, $d$ and $e$ be distinct positive real numbers such that $a^2 + b^2 + c^2 + d^2 + e^2 = ab + ac + ad + ae + bc + bd + be + cd + ce + de$ $a)$ Prove that among these $5$ numbers there exists triplet such that they cannot be sides of a triangle $b)$ Prove that, for $a)$, there exists at least $6$ different triplets