Bosnia and Herzegovina European Girls Mathematical Olympiad TST 2018

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1 $a)$ Prove that there exists 5 nonnegative real numbers with sum equal to 1, such that no matter how we arrange them on circle, two neighboring numbers exist with product not less than $\frac{1}{9}$
$a)$ Prove that for every 5 nonnegative real numbers with sum equal to 1, we can arrange them on circle, such that product of every two neighboring numbers is not greater than $\frac{1}{9}$

2 Prove that for every pair of positive integers (m,n), bigger than 2, there exists positive integer k and numbers $a_0, a_1, ..., a_k$, which are bigger than 2, such that $a_0 = m$, $a_1 = n$ and for all $i = 0, 1, ..., k-1$ holds

$$a_i + a_{i+1} \mid a_i a_{i+1} + 1$$

3 Let O be a circumcenter of acute triangle ABC and let O_1 and O_2 be circumcenters of triangles OAB and OAC, respectively. Circumcircles of triangles OAB and OAC intersect side BC in points D $(D \neq B)$ and E $(E \neq C)$, respectively. Perpendicular bisector of side BC intersects side AC in point $F(F \neq A)$. Prove that circumcenter of triangle ADE lies on AC iff F lies on line O_1O_2

4 It is given positive integer n. Let $a_1, a_2, ..., a_n$ be positive integers with sum $2S$, $S \in \mathbb{N}$. Positive integer k is called separator if you can pick k different indices $i_1, i_2, ..., i_k$ from set $\{1, 2, ..., n\}$ such that $a_{i_1} + a_{i_2} + ... + a_{i_k} = S$. Find, in terms of n, maximum number of separators