5th IGO

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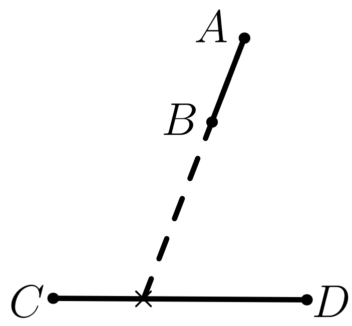
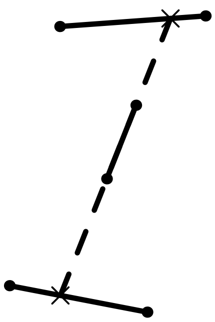
by parmenides51, bgn

–	Elementary
1	As shown below, there is a 40×30 paper with a filled 10×5 rectangle inside of it. We want to cut out the filled rectangle from the paper using four straight cuts. Each straight cut is a straight line that divides the paper into two pieces, and we keep the piece containing the filled rectangle. The goal is to minimize the total length of the straight cuts. How to achieve this goal, and what is that minimized length? Show the correct cuts and write the final answer. There is no need to prove the answer. *Proposed by Morteza Saghafian*
2	Convex hexagon $A_1A_2A_3A_4A_5A_6$ lies in the interior of convex hexagon $B_1B_2B_3B_4B_5B_6$ such that $A_1A_2 \parallel B_1B_2$, $A_2A_3 \parallel B_2B_3$,..., $A_6A_1 \parallel B_6B_1$. Prove that the areas of simple hexagons $A_1B_2A_3B_4A_5B_6$ and $B_1A_2B_3A_4B_5A_6$ are equal. (A simple hexagon is a hexagon which does not intersect itself.) *Proposed by Hirad Aalipanah - Mahdi Etesamifard*
3	In the given figure, $ABCD$ is a parallelogram. We know that $\angle D = 60^\circ$, $AD = 2$ and $AB = \sqrt{3}+1$. Point M is the midpoint of AD. Segment CK is the angle bisector of C. Find the angle CKB. *Proposed by Mahdi Etesamifard*
4	There are two circles with centers O_1, O_2 lie inside of circle ω and are tangent to it. Chord AB of ω is tangent to these two circles such that they lie on opposite sides of this chord. Prove that $\angle O_1AO_2 + \angle O_1BO_2 > 90^\circ$. *Proposed by Iman Maghsoudi*
5	There are some segments on the plane such that no two of them intersect each other (even at the ending points). We say segment AB **breaks** segment CD if the extension of AB cuts CD at some point between C and D.

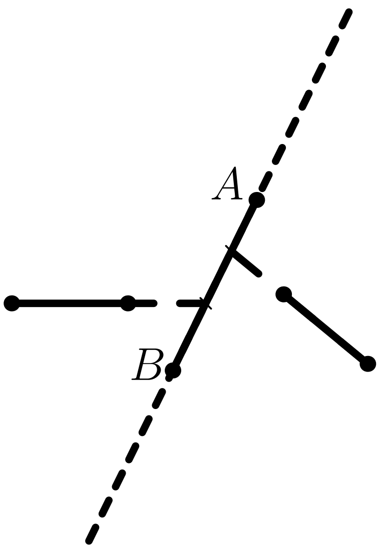


$a)$ Is it possible that each segment when extended from both ends, breaks exactly one other segment from each way?



$b)$ A segment is called **surrounded** if from both sides of it, there is exactly one segment that breaks it.

(*e.g.* segment AB in the figure.) Is it possible to have all segments to be surrounded?



Proposed by Morteza Saghafian

– Intermediate

1 There are three rectangles in the following figure. The lengths of some segments are shown. Find the length of the segment XY .
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OOQCK4BGAYYCw/s320/igo%2B2018%2Bintermediate%2Bp1.png
Proposed by Hirad Aalipanah

2 In convex quadrilateral $ABCD$, the diagonals AC and BD meet at the point P. We know that $\angle DAC = 90^o$ and $2\angle ADB = \angle ACB$. If we have $\angle DBC + 2\angle ADC = 180^o$ prove that $2AP = BP$.

Proposed by Iman Maghsoudi

3 Let ω_1, ω_2 be two circles with centers O_1 and O_2, respectively. These two circles intersect each other at points A and B. Line O_1B intersects ω_2 for the second time at point C, and line O_2A intersects ω_1 for the second time at point D . Let X be the second intersection of AC and ω_1. Also Y is the second intersection point of BD and ω_2. Prove that $CX = DY$.

Proposed by Alireza Dadgarnia

4 We have a polyhedron all faces of which are triangle. Let P be an arbitrary point on one of the edges of this polyhedron such that P is not the midpoint or endpoint of this edge. Assume that $P_0 = P$. In each step, connect P_i to the centroid of one of the faces containing it. This line meets

the perimeter of this face again at point P_{i+1}. Continue this process with P_{i+1} and the other face containing P_{i+1}. Prove that by continuing this process, we cannot pass through all the faces. (The centroid of a triangle is the point of intersection of its medians.)

Proposed by Mahdi Etesamifard - Morteza Saghafian

5 Suppose that $ABCD$ is a parallelogram such that $\angle DAC = 90^o$. Let H be the foot of perpendicular from A to DC, also let P be a point along the line AC such that the line PD is tangent to the circumcircle of the triangle ABD. Prove that $\angle PBA = \angle DBH$.

Proposed by Iman Maghsoudi

– Advanced

1 Two circles ω_1, ω_2 intersect each other at points A, B. Let PQ be a common tangent line of these two circles with $P \in \omega_1$ and $Q \in \omega_2$. An arbitrary point X lies on ω_1. Line AX intersects ω_2 for the second time at Y . Point $Y' \neq Y$ lies on ω_2 such that $QY = QY'$. Line $Y'B$ intersects ω_1 for the second time at X'. Prove that $PX = PX'$.

Proposed by Morteza Saghafian

2 In acute triangle $ABC, \angle A = 45^o$. Points O, H are the circumcenter and the orthocenter of ABC, respectively. D is the foot of altitude from B. Point X is the midpoint of arc AH of the circumcircle of triangle ADH that contains D. Prove that $DX = DO$.

Proposed by Fatemeh Sajadi

3 Find all possible values of integer $n > 3$ such that there is a convex n-gon in which, each diagonal is the perpendicular bisector of at least one other diagonal.

Proposed by Mahdi Etesamifard

4 Quadrilateral $ABCD$ is circumscribed around a circle. Diagonals AC, BD are not perpendicular to each other. The angle bisectors of angles between these diagonals, intersect the segments AB, BC, CD and DA at points K, L, M and N. Given that $KLMN$ is cyclic, prove that so is $ABCD$.

Proposed by Nikolai Beluhov (Bulgaria)

5 $ABCD$ is a cyclic quadrilateral. A circle passing through A, B is tangent to segment CD at point E. Another circle passing through C, D is tangent to AB at point F. Point G is the intersection point of AE, DF, and point H is the intersection point of BE, CF. Prove that the incenters of triangles AGF, BHF, CHE, DGE lie on a circle.

Proposed by Le Viet An (Vietnam)