Bosnia and Herzegovina Team Selection Test 2001

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–	Day 1
1	On circle there are points A, B and C such that they divide circle in ratio $3:5:7$. Find angles of triangle ABC
2	For positive integers x, y and z holds $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{z^2}$. Prove that $xyz \geq 3600$
3	Find maximal value of positive integer n such that there exists subset of $S=\{1,2,...,2001\}$ with n elements, such that equation $y=2x$ does not have solutions in set $S \times S$
–	Day 2
4	In plane there are two circles with radiuses r_1 and r_2, one outside the other. There are two external common tangents on those circles and one internal common tangent. The internal one intersects external ones in points A and B and touches one of the circles in point C. Prove that $AC \cdot BC = r_1 \cdot r_2$
5	Let n be a positive integer, $n \geq 1$ and $x_1,x_2,...,x_n$ positive real numbers such that $x_1+x_2+...+x_n=1$. Does the following inequality hold $$\sum_{i=1}^{n} \frac{x_i}{1-x_1 \cdot ... \cdot x_{i-1} \cdot x_{i+1} \cdot ... x_n} \leq \frac{1}{1-\left(\frac{1}{n}\right)^{n-1}}$$
6	Prove that there exists infinitely many positive integers n such that equation $(x+y+z)^3=n^2xyz$ has solution (x,y,z) in set $\mathbb{N}^3$