Bosnia and Herzegovina Team Selection Test 2000

www.artofproblemsolving.com/community/c732710

by gobathegreat

– Day 1

1 Find real roots x_1, x_2 of equation $x^5-55x+21=0$, if we know $x_1 \cdot x_2=1$

2 Let S be a point inside triangle ABC and let lines AS, BS and CS intersect sides BC, CA and AB in points X, Y and Z, respectively. Prove that

$$\frac{BX \cdot CX}{AX^2}+\frac{CY \cdot AY}{BY^2}+\frac{AZ \cdot BZ}{CZ^2}=\frac{R}{r}-1$$

iff S is incenter of ABC

3 We call *Pythagorean triple* a triple (x,y,z) of positive integers such that $x<y<z$ and $x^2+y^2=z^2$. Prove that for all $n \in \mathbb{N}$ number 2^{n+1} is in exactly n *Pythagorean triples*

– Day 2

4 Prove that for all positive real a, b and c holds:

$$\frac{bc}{a^2+2bc}+\frac{ac}{b^2+2ac}+\frac{ab}{c^2+2ab} \leq 1 \leq \frac{a^2}{a^2+2bc}+\frac{b^2}{b^2+2ac}+\frac{c^2}{c^2+2ab}$$

5 Let T_m be a number of non-congruent triangles which perimeter is m and all its sides are positive integers. Prove that: $a)$ $T_{1999} > T_{2000}$ $b)$ $T_{4n+1}=T_{4n-2}+n$, $(n \in \mathbb{N})$

6 It is given triangle ABC such that $\angle ABC=3\angle CAB$. On side AC there are two points M and N in order $A-N-M-C$ and $\angle CBM=\angle MBN=\angle NBA$. Let L be an arbitrary point on side BN and K point on BM such that $LK \mid\mid AC$. Prove that lines AL, NK and BC are concurrent