Bosnia and Herzegovina Team Selection Test 1996

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by gobathegreat

– Day 1

1 $a)$ Let a, b and c be positive real numbers. Prove that for all positive integers m holds:

$$(a+b)^m+(b+c)^m+(c+a)^m \leq 2^m(a^m+b^m+c^m)$$

$b)$ Does inequality $a)$ holds for
$1)$ arbitrary real numbers a, b and c
$2)$ any integer m

2 $a)$ Let m and n be positive integers. If $m>1$ prove that $n \mid \phi(m^n-1)$ where ϕ is Euler function $b)$ Prove that number of elements in sequence $1,2,...,n$ $(n \in \mathbb{N})$, which greatest common divisor with n is d, is $\phi\left(\frac{n}{d}\right)$

3 Let M be a point inside quadrilateral $ABCD$ such that $ABMD$ is parallelogram. If $\angle CBM = \angle CDM$ prove that $\angle ACD = \angle BCM$

– Day 2

4 Solve the functional equation

$$f(x+y)+f(x-y)=2f(x)\cos{y}$$

where $x,y \in \mathbb{R}$ and $f : \mathbb{R} \rightarrow \mathbb{R}$

5 Group of 10 people are buying books. We know the following: $i)$ Every person bought four different books $ii)$ Every two persons bought at least one book common for both of them
Taking in consideration book which was bought by maximum number of people, determine minimal value of that number

6 Let a and b be two integers which are coprime and let n be one variable integer. Determine probability that number of solutions (x,y), where x and y are nonnegative integers, of equation $ax+by=n$ is $\left\lfloor \frac{n}{ab} \right\rfloor + 1$