Bosnia and Herzegovina Team Selection Test 1998

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–	Day 1
1	Let P_1, P_2, P_3, P_4 and P_5 be five different points which are inside D or on the border of figure D. Let $M = min\{P_iP_j \mid i \neq j\}$ be minimal distance between different points P_i. For which configuration of points P_i, value M is at maximum, if : $a)$ D is unit square $b)$ D is equilateral triangle with side equal 1 $c)$ D is unit circle, circle with radius 1
2	For positive real numbers x, y and z holds $x^2+y^2+z^2=1$. Prove that $$\frac{x}{1+x^2}+\frac{y}{1+y^2}+\frac{z}{1+z^2} \leq \frac{3\sqrt{3}}{4}$$
3	Angle bisectors of angles by vertices A, B and C in triangle ABC intersect opposing sides in points A_1, B_1 and C_1, respectively. Let M be an arbitrary point on one of the lines A_1B_1, B_1C_1 and C_1A_1. Let M_1, M_2 and M_3 be orthogonal projections of point M on lines BC, CA and AB, respectively. Prove that one of the lines MM_1, MM_2 and MM_3 is equal to sum of other two
–	Day 2
4	Circle k with radius r touches the line p in point A. Let AB be a dimeter of circle and C an arbitrary point of circle distinct from points A and B. Let D be a foot of perpendicular from point C to line AB. Let E be a point on extension of line CD, over point D, such that $ED=BC$. Let tangents on circle from point E intersect line p in points K and N. Prove that length of KN does not depend from C
5	Let a, b and c be integers such that $$bc+ad=1$$ $$ac+2bd=1$$ Prove that $a^2+c^2=2b^2+2d^2$
6	Sequence of integers $\{u_n\}_{n \in \mathbb{N}_0}$ is given as: $u_0=0$, $u_{2n}=u_n$, $u_{2n+1}=1-u_n$ for all $n \in \mathbb{N}_0$ $a)$ Find u_{1998} $b)$ If p is a positive integer and $m=(2^p-1)^2$, find u_m