Regional Olympiad - Federation of Bosnia and Herzegovina 2014

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- Sarajevo, April 26th

- Grade 9

1 Find all possible values of

$$\frac{(a+b-c)^2}{(a-c)(b-c)} + \frac{(b+c-a)^2}{(b-a)(c-a)} + \frac{(c+a-b)^2}{(c-b)(a-b)}$$

2 Solve the equation, where x and y are positive integers:

$$x^3 - y^3 = 999$$

3 In triangle ABC ($b \geq c$), point E is the midpoint of shorter arc BC. If D is the point such that ED is the diameter of circumcircle ABC, prove that $\angle DEA = \frac{1}{2}(\beta - \gamma)$

4 Determine the set S with minimal number of points defining 7 distinct lines

- Grade 10

1 Solve the equation:

$$\frac{xy}{z} + \frac{yz}{x} + \frac{zx}{y} = 3$$

where x, y and z are integers

2 Let a, b and c be positive real numbers such that $ab + bc + ca = 1$. Prove the inequality:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 3(a+b+c)$$

3 Let $ABCD$ be a parallelogram. Let M be a point on the side AB and N be a point on the side BC such that the segments AM and CN have equal lengths and are non-zero. The lines AN and CM meet at Q.
Prove that the line DQ is the bisector of the angle $\angle ADC$.

Alternative formulation. Let $ABCD$ be a parallelogram. Let M and N be points on the sides AB and BC, respectively, such that $AM = CN \neq 0$. The lines AN and CM intersect at a point Q.

Prove that the point Q lies on the bisector of the angle $\angle ADC$.

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4   How namy subsets with 3 elements of set $S = \{1, 2, 3, ..., 19, 20\}$ exist, such that their product is divisible by 4.

–   Grade 11

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1   Solve logarithmical equation $x^{\log_3 x - 1} + 2(x - 1)^{\log_3 x} = 3x^2$

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2   Solve the equation

$$x^2 + y^2 + z^2 = 686$$

where x, y and z are positive integers

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3   Excircle of triangle ABC to side AB of triangle ABC touches side AB in point D. Determine ratio $AD : BD$ if $\angle CAB = 2\angle ADC$

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4   At the beginning of school year in one of the first grade classes: $i)$ every student had exatly 20 acquaintances $ii)$ every two students knowing each other had exactly 13 mutual acquaintances $iii)$ every two students not knowing each other had exactly 12 mutual acquaintances Find number of students in this class

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–   Grade 12

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1   Find all real solutions of the equation:

$$x = \frac{2z^2}{1 + z^2}$$

$$y = \frac{2x^2}{1 + x^2}$$

$$z = \frac{2y^2}{1 + y^2}$$

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2   Problem 3 for grade 11

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3   Find all integers n such that $n^4 - 8n + 15$ is product of two consecutive integers

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4   Problem 4 for grade 11