Regional Olympiad - Federation of Bosnia and Herzegovina 2013

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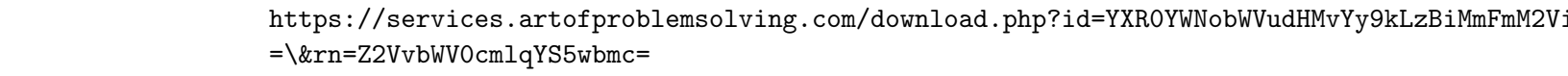
by gobathegreat

–	Sarajevo, April 28th
–	Grade 9
1	If x and y are real numbers such that $x^{2013}+y^{2013}>x^{2012}+y^{2012}$, prove that $x^{2014}+y^{2014}>x^{2013}+y^{2013}$
2	In triangle ABC, $\angle ACB=50^\circ$ and $\angle CBA=70^\circ$. Let D be a foot of perpendicular from point A to side BC, O circumcenter of ABC and E antipode of A in circumcircle ABC. Find $\angle DAE$
3	Find maximal positive integer p such that 5^7 is sum of p consecutive positive integers
4	$a)$ Is it possible, on modified chessboard 20×30, to draw a line which cuts exactly 50 cells where chessboard cells are squares 1×1 $b)$ What is the maximum number of cells which line can cut on chessboard $m\times n$, $m,n\in\mathbb{N}$
–	Grade 10
1	If x and y are nonnegative real numbers such that $x+y=1$, determine minimal and maximal value of $$A=x\sqrt{1+y}+y\sqrt{1+x}$$
2	In circle with radius 10, point M is on chord PQ such that $PM=5$ and $MQ=10$. Through point M we draw chords AB and CD, and points X and Y are intersection points of chords AD and BC with chord PQ (see picture), respectively. If $XM=3$ find MY 
3	Find all integers a such that $\sqrt{\frac{9a+4}{a-6}}$ is rational number
4	Problem 4 for grade 9
–	Grade 11

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1 Let a and b be real numbers from interval $\left[0, \frac{\pi}{2}\right]$. Prove that

$$\sin^6 a + 3\sin^2 a \cos^2 b + \cos^6 b = 1$$

if and only if $a = b$

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2 Find all integers a, b, c and d such that

$$a^2 + 5b^2 - 2c^2 - 2cd - 3d^2 = 0$$

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3 Convex quadrilateral is divided by diagonals into four triangles with congruent inscribed circles. Prove that this quadrilateral is rhombus.

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4 If $A = \{1, 2, ..., 4s-1, 4s\}$ and $S \subseteq A$ such that $\mid S \mid = 2s + 2$, prove that in S we can find three distinct numbers x, y and z such that $x + y = 2z$

– Grade 12

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1 If a, b and c are nonnegative real numbers such that $a^2 + b^2 + c^2 = 1$, prove that

$$\frac{1}{2} \leq \frac{a}{1+a^4} + \frac{b}{1+b^4} + \frac{c}{1+c^4} \leq \frac{9\sqrt{3}}{10}$$

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2 If x and y are real numbers, prove that $\frac{4x^2+1}{y^2+2}$ is not integer

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3 Problem 3 for grade 11

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4 Problem 4 for grade 11

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