Regional Olympiad - Federation of Bosnia and Herzegovina 2012

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– Zenica, April 14th

– Grade 9

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1 Find all possible values of

$$\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}+\frac{1}{b+c}\right)+\frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{c+a}\right)+\frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{a+b}\right)-\frac{1}{a+b+c}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{a+b}+\frac{1}{b+c}+\right.$$

where a, b and c are positive real numbers such that $ab+bc+ca=abc$

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2 Let a, b, c, d, e, f and g be seven distinct positive integers not bigger than 7. Find all primes which can be expressed as $abcd+efg$

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3 Find remainder when dividing upon 2012 number

$$A=1\cdot 2+2\cdot 3+3\cdot 4+...+2009\cdot 2010+2010\cdot 2011$$

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4 Let S be an incenter of triangle ABC and let incircle touch sides AC and AB in points P and Q, respectively. Lines BS and CS intersect line PQ in points M and N, respectively. Prove that points M, N, B and C are concyclic

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– Grade 10

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1 Solve equation

$$x^2-\sqrt{a-x}=a$$

where x is real number and a is real parameter

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2 Harry Potter can do any of the three tricks arbitrary number of times: $i)$ switch 1 plum and 1 pear with 2 apples $ii)$ switch 1 pear and 1 apple with 3 plums $iii)$ switch 1 apple and 1 plum with 4 pears
In the beginning, Harry had 2012 of plums, apples and pears, each. Harry did some tricks and now he has 2012 apples, 2012 pears and more than 2012 plums. What is the minimal number of plums he can have?

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3 Quadrilateral $ABCD$ is cyclic. Line through point D parallel with line BC intersects CA in point P, line AB in point Q and circumcircle of $ABCD$ in point R. Line through point D parallel with line AB intersects AC in point S, line BC in point T and circumcircle of $ABCD$ in point U. If $PQ=QR$, prove that $ST=TU$

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4 Can number 2012^n-3^n be perfect square, while n is positive integer

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– Grade 11

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1 For which real numbers x and α inequality holds:

$$\log_2 x + \log_x 2 + 2\cos\alpha \leq 0$$

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2 On football toornament there were 4 teams participating. Every team played exactly one match with every other team. For the win, winner gets 3 points, while if draw both teams get 1 point. If at the end of tournament every team had different number of points and first place team had 6 points, find the points of other teams

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3 Prove tha number $19 \cdot 8^n + 17$ is composite for every positive integer n

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4 In triangle ABC point O is circumcenter. Point T is centroid of ABC, and points D, E and F are circumcenters of triangles TBC, TCA and TAB. Prove that O is centroid of DEF

– Grade 12

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1 Problem 2 for grade 11

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2 Let a, b, c and d be integers such that ac, bd and $bc+ad$ are divisible with positive integer m. Show that numbers bc and ad are divisible with m

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3 Problem 4 for grade 11

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4 Prove the inequality:

$$\frac{A+a+B+b}{A+a+B+b+c+r} + \frac{B+b+C+c}{B+b+C+c+a+r} > \frac{C+c+A+a}{C+c+A+a+b+r}$$

where A, B, C, a, b, c and r are positive real numbers
