Regional Olympiad - Federation of Bosnia and Herzegovina 2011

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by gobathegreat

–	Sarajevo, April 16th
–	Grade 9
1	Factorise $$(a+2b-3c)^3+(b+2c-3a)^3+(c+2a-3b)^3$$
2	At the round table there are 10 students. Every of the students thinks of a number and says that number to its immediate neighbors (left and right) such that others do not hear him. So every student knows three numbers. After that every student publicly says arithmetic mean of two numbers he found out from his neghbors. If those arithmetic means were 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10, respectively, which number thought student who told publicly number 6
3	Triangle AOB is rotated in plane around point O for 90° and it maps in triangle A_1OB_1 (A maps to A_1, B maps to B_1). Prove that median of triangle OAB_1 of side AB_1 is orthogonal to A_1B
4	For positive integer n, prove that at least one of the numbers $$A=2n-1, B=5n-1, C=13n-1$$ is not perfect square
–	Grade 10
1	Find the real number coefficient c of polynomial x^2+x+c, if his roots x_1 and x_2 satisfy following: $$\frac{2x_1^3}{2+x_2}+\frac{2x_2^3}{2+x_1}=-1$$
2	If $p>2$ is prime number and m and n are positive integers such that $$\frac{m}{n}=1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{p-1}$$ Prove that p divides m

3 Let I be the incircle and O a circumcenter of triangle ABC such that $\angle ACB = 30^{\circ}$. On sides AC and BC there are points E and D, respectively, such that $EA = AB = BD$. Prove that $DE = IO$ and $DE \perp IO$

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4 Let n be a positive integer and set $S = \{n, n+1, n+2, ..., 5n\}$ $a)$ If set S is divided into two disjoint sets , prove that there exist three numbers x, y and z(possibly equal) which belong to same subset of S and $x + y = z$ $b)$ Does $a)$ hold for set $S = \{n, n+1, n+2, ..., 5n-1\}$

– Grade 11

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1 Determine value of real parameter λ such that equation

$$\frac{1}{\sin x} + \frac{1}{\cos x} = \lambda$$

has root in interval $(0, \frac{\pi}{2})$

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2 For positive integers a and b holds $a^3 + 4a = b^2$. Prove that $a = 2t^2$ for some positive integer t

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3 Let AD and BE be angle bisectors in triangle ABC. Let x, y and z be distances from point M, which lies on segment DE, from sides BC, CA and AB, respectively. Prove that $z = x + y$

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4 Prove that among any 6 irrational numbers you can pick three numbers a, b and c such that numbers $a + b$, $b + c$ and $c + a$ are irrational

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– Grade 12

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1 Problem 3 for grade 11

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2 If for real numbers x and y holds $\left(x + \sqrt{1 + y^2}\right)\left(y + \sqrt{1 + x^2}\right) = 1$ prove that

$$\left(x + \sqrt{1 + x^2}\right)\left(y + \sqrt{1 + y^2}\right) = 1$$

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3 If n is a positive integer and $n + 1$ is divisible with 24, prove that sum of all positive divisors of n is divisible with 24

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4 Problem 4 for grade 11
