Regional Olympiad - Federation of Bosnia and Herzegovina 2009

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– Grade 9

1 Find all triplets of integers (x,y,z) such that

$$xy(x^2-y^2)+yz(y^2-z^2)+zx(z^2-x^2)=1$$

2 Find minimum of $x+y+z$ where x, y and z are real numbers such that $x \geq 4$, $y \geq 5$, $z \geq 6$ and $x^2+y^2+z^2 \geq 90$

3 Is it possible in a plane mark 10 red, 10 blue and 10 green points (all distinct) such that three conditions hold: $i)$ For every red point A there exists a blue point closer to point A than any other green point $ii)$ For every blue point B there exists a green point closer to point B than any other red point $iii)$ For every green point C there exists a red point closer to point C than any other blue point

4 Let C be a circle with center O and radius R. From point A of circle C we construct a tangent t on circle C. We construct line d through point O whch intersects tangent t in point M and circle C in points B and D (B lies between points O and M). If $AM=R\sqrt{3}$, prove: $a)$ Triangle AMD is isosceles $b)$ Circumcenter of AMD lies on circle C

– Grade 10

1 In triangle ABC such that $\angle ACB=90^{\circ}$, let point H be foot of perpendicular from point C to side AB. Show that sum of radiuses of incircles of ABC, BCH and ACH is CH

2 Find minimal value of $a \in \mathbb{R}$ such that system

$$\sqrt{x-1}+\sqrt{y-1}+\sqrt{z-1}=a-1$$
$$\sqrt{x+1}+\sqrt{y+1}+\sqrt{z+1}=a+1$$

has solution in set of real numbers

3 Decomposition of number n is showing n as a sum of positive integers (not neccessarily distinct). Order of addends is important. For every positive integer n show that number of decompositions is 2^{n-1}

4 Let x and y be positive integers such that $\frac{x^2-1}{y+1}+\frac{y^2-1}{x+1}$ is integer. Prove that numbers $\frac{x^2-1}{y+1}$ and $\frac{y^2-1}{x+1}$ are integers

– Grade 11

1 In triangle ABC holds $\angle ACB=90^{\circ}$, $\angle BAC=30^{\circ}$ and $BC=1$. In triangle ABC is inscribed equilateral triangle (every side of a triangle ABC contains one vertex of inscribed triangle). Find the least possible value of side of inscribed equilateral triangle

2 For given positive integer n find all quartets (x_1,x_2,x_3,x_4) such that $x_1^2+x_2^2+x_3^2+x_4^2=4^n$

3 There are n positive integers on the board. We can add only positive integers $c=\frac{a+b}{a-b}$, where a and b are numbers already writted on the board. $a)$ Find minimal value of n, such that with adding numbers with described method, we can get any positive integer number written on the board $b)$ For such n, find numbers written on the board at the beginning

4 What is the minimal value of $\sqrt{2x+1}+\sqrt{3y+1}+\sqrt{4z+1}$, if x, y and z are nonnegative real numbers such that $x+y+z=4$

– Grade 12

1 Prove that for every positive integer m there exists positive integer n such that $m+n+1$ is perfect square and $mn+1$ is perfect cube of some positive integers

2 Let ABC be an equilateral triangle such that length of its altitude is 1. Circle with center on the same side of line AB as point C and radius 1 touches side AB. Circle rolls on the side AB. While the circle is rolling, it constantly intersects sides AC and BC. Prove that length of an arc of the circle, which lies inside the triangle, is constant

3 Problem 3 for grade 11

4 Problem 4 for grade 11