National Math Olympiad (Second Round) 2015

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Day 1 April 7th

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1 Consider a cake in the shape of a circle. It's been divided to some inequal parts by its radii. Arash and Bahram want to eat this cake. At the very first, Arash takes one of the parts. In the next steps, they consecutively pick up a piece adjacent to another piece formerly removed. Suppose that the cake has been divided to 5 parts. Prove that Arash can choose his pieces in such a way at least half of the cake is his.

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2 There's a special computer and it has a memory. At first, it's memory just contains x.
We fill up the memory with the following rules.

1) If $f \neq 0$ is in the memory, then we can also put $\frac{1}{f}$ in it.

2) If f, g are in the memory, then we can also put $f+g$ and $f-g$ in it.

Find all natural number n such that we can have x^n in the memory.

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3 Consider a triangle ABC . The points D, E are on sides AB, AC such that $BDEC$ is a cyclic quadrilateral. Let P be the intersection of BE and CD. H is a point on AC such that $\angle PHA = 90^\circ$. Let M, N be the midpoints of AP, BC. Prove that: $ACD \sim MNH$.

Day 2 April 8th

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1 In quadrilateral $ABCD$, AC is bisector of $\hat{A}$ and $\widehat{ADC} = \widehat{ACB}$. X and Y are feet of perpendicular from A to BC and CD,respectively.Prove that orthocenter of triangle AXY is on BD.

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2 A circle is divided into $2n$ equal by $2n$ points. Ali draws $n+1$ arcs, of length $1, 2, \ldots, n+1$. Prove that we can find two arcs, such that one of them is inside in the other one.

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3 Let $n \geq 50$ be a natural number. Prove that n is expressible as sum of two natural numbers $n = x+y$, so that for every prime number p such that $p \mid x$ or $p \mid y$ we have $\sqrt{n} \geq p$. For example for $n = 94$ we have $x = 80, y = 14$.
