www.artofproblemsolving.com/community/c797971
by XbenX

– In a society some people know each other. For a positive integer $k \geq 3$ we say the society is *k-good* , if any k-tuple of people can be seated in a round table , so that any two neighbours know each other.
Prove that , if a society is *6-good* , then it is also *7-good* .

Proposed by Josef Tkadlec

– Real numbers x, y, z are chosen such that

$$\frac{1}{|x^2+2yz|}, \frac{1}{|y^2+2zx|}, \frac{1}{|x^2+2xy|}$$

are lengths of a non-degenerate triangle .
Find all possible values of $xy + yz + zx$.

Proposed by Michael Rolnek

– In $\triangle ABC$,let D be the intersection of angle bisector of $\angle BAC$ and BC .
Let E, F be the circumcenters of $\triangle ABD, \triangle ACD$,respectively.
What is the size of $\angle BAC$ if the circumcenter of $\triangle AEF$ lies on the line BC .

Proposed by Patrik Bak

– Let a, b, c be a triple of integers , which are triangle sides , and do not have a common divisor greater than 1, for wich all three

$$\frac{a^2+b^2-c^2}{a+b-c}, \frac{b^2+c^2-a^2}{b+c-a}, \frac{c^2+a^2-b^2}{c+a-b}$$

are integers.
Prove that the product of the three denominators or it's double is a perfect square .

Proposed by Jaromr imia

– An isosceles trapezium $ABCD$ is given , such that AB is the longest base.
Let I be the incenter of of $\triangle ABC$,and J be the C-excenter of $\triangle ACD$.
Prove that $IJ \parallel AB$.

Proposed by Patrik Bak

– Find the smallest natural number n such that for any coloration of the numbers $1, 2, \dots n$ with three different colors, there exists 2 numbers of the same color, whose difference is a perfect square.

Proposed by Vojitech Blint , Michael Rolnek, Josef Tkadlec