Bosnia Herzegovina Team Selection Test 2015

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by gobathegreat

–	Day 1
1	Determine the minimum value of the expression $$\frac{a+1}{a(a+2)}+\frac{b+1}{b(b+2)}+\frac{c+1}{c(c+2)}$$ for positive real numbers a, b, c such that $a+b+c \leq 3$.
2	Let D be an arbitrary point on side AB of triangle ABC. Circumcircles of triangles BCD and ACD intersect sides AC and BC at points E and F, respectively. Perpendicular bisector of EF cuts AB at point M, and line perpendicular to AB at D at point N. Lines AB and EF intersect at point T, and the second point of intersection of circumcircle of triangle CMD and line TC is U. Prove that $NC = NU$
3	Prove that there exist infinitely many composite positive integers n such that n divides $3^{n-1} - 2^{n-1}$.
–	Day 2
4	Let X be a set which consists from 8 consecutive positive integers. Set X is divided on two disjoint subsets A and B with equal number of elements. If sum of squares of elements from set A is equal to sum of squares of elements from set B, prove that sum of elements of set A is equal to sum of elements of set B.
5	Let N be a positive integer. It is given set of weights which satisfies following conditions: $i)$ Every weight from set has some weight from $1, 2, ..., N$; $ii)$ For every $i \in 1, 2, ..., N$ in given set there exists weight i; $iii)$ Sum of all weights from given set is even positive integer. Prove that set can be partitioned into two disjoint sets which have equal weight
6	Let D, E and F be points in which incircle of triangle ABC touches sides BC, CA and AB, respectively, and let I be a center of that circle.Furthermore, let P be a foot of perpendicular from point I to line AD, and let M be midpoint of DE. If $\{N\} = PM \cap AC$, prove that $DN \parallel EF$