Serbia National Math Olympiad 2014
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by mihajlon

– Day 1

1 Determine all functions $f:\mathbb{R}\rightarrow\mathbb{R}$ such that for all x, $y \in \mathbb{R}$ hold:

$$f(xf(y)-yf(x))=f(xy)-xy$$

Proposed by Dusan Djukic

2 On sides BC and AC of $\triangle ABC$ given are D and E, respectively. Let F ($F \neq C$) be a point of intersection of circumcircle of $\triangle CED$ and line that is parallel to AB and passing through C. Let G be a point of intersection of line FD and side AB, and let H be on line AB such that $\angle HDA = \angle GEB$ and $H-A-B$. If $DG = EH$, prove that point of intersection of AD and BE lie on angle bisector of $\angle ACB$.

Proposed by Milos Milosavljevic

3 Two players are playing game. Players alternately write down one natural number greater than 1, but it is not allowed to write linear combination previously written numbers with nonnegative integer coefficients. Player lose a game if he can't write a new number. Does any of players can have wiining strategy, if yes, then which one of them?

Journal "Kvant" / Aleksandar Ilic

– Day 2

4 We call natural number n [i]$crazy$[/i] iff there exist natural numbers a, $b > 1$ such that $n = a^b + b$. Whether there exist 2014 consecutive natural numbers among which are 2012 [i]$crazy$[/i] numbers?

Proposed by Milos Milosavljevic

5 Regular n-gon is divided to triangles using $n-3$ diagonals of which none of them have common points with another inside polygon. How much among this triangles can there be the most not congruent?

Proposed by Dusan Djukic

6 In a triangle ABC, let D and E be the feet of the angle bisectors of angles A and B, respectively. A rhombus is inscribed into the quadrilateral $AEDB$ (all vertices of the rhombus

lie on different sides of $AEDB$). Let φ be the non-obtuse angle of the rhombus. Prove that $\varphi \le \max\{\angle BAC, \angle ABC\}$

[i]Proposed by Dusan Djukic $IMO\ Shortlist\ 2013$[/i]