The problems from the CCA Math Bonanza held on 2/2/2018

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by trumpeter

–	Individual Round
I1	Consider the arithmetic sequence of integers with first term -7 and second term 17. What is the sum of the squares of the first three terms of the sequence? *2019 CCA Math Bonanza Individual Round#1*
I2	Square 1 is drawn with side length 4. Square 2 is then drawn inside of Square 1, with its vertices at the midpoints of the sides of Square 1. Given Square n for a positive integer n, we draw Square $n+1$ with vertices at the midpoints of the sides of Square n. For any positive integer n, we draw Circle n through the four vertices of Square n. What is the area of Circle 7? *2019 CCA Math Bonanza Individual Round#2*
I3	Sristan Thin is walking around the Cartesian plane. From any point (x, y), Sristan can move to $(x+1, y)$ or $(x+1, y+3)$. How many paths can Sristan take from $(0, 0)$ to $(9, 9)$? *2019 CCA Math Bonanza Individual Round#3*
I4	How many ordered pairs (a, b) of positive integers are there such that $$\gcd(a,b)^3 = \operatorname{lcm}(a,b)^2 = 4^6$$ is true? *2019 CCA Math Bonanza Individual Round#4*
I5	How many ways are there to rearrange the letters of CCAMB such that at least one C comes before the A? *2019 CCA Math Bonanza Individual Round#5*
I6	If distinct digits D, E, L, M, Q (between 0 and 9 inclusive) satisfy $\begin{array}{r} E\ L \\ +\ M\ E\ M \\ \hline Q\ E\ D \end{array}$ what is the maximum possible value of the three digit integer QED? *2019 CCA Math Bonanza Individual Round#6*
I7	How many permutations π of $\{1, 2, \ldots, 7\}$ are there such that $\pi(k) \leq 2k$ for $k = 1, \ldots, 7$? A permutation π of a set S is a function from S to itself such that if $a \neq b$, then $\pi(a) \neq \pi(b)$. For example, $\pi(x) = x$ and $\pi(x) = 8 - x$ are permutations of $\{1, 2, \ldots, 7\}$ but $\pi(x) = 1$ is not.

2019 CCA Math Bonanza Individual Round#7

I8 If $a! + (a+2)!$ divides $(a+4)!$ for some nonnegative integer a, what are all possible values of a?

2019 CCA Math Bonanza Individual Round#8

I9 Isosceles triangle $\triangle ABC$ has $\angle BAC = \angle ABC = 30^\circ$ and $AC = BC = 2$. If the midpoints of BC and AC are M and N, respectively, and the circumcircle of $\triangle CMN$ meets AB at D and E with D closer to A than E is, what is the area of $MNDE$?

2019 CCA Math Bonanza Individual Round#9

I10 What is the minimum possible value of

$$|x| - |x-1| + |x+2| - |x-3| + |x+4| - \cdots - |x-2019|$$

over all real x?

2019 CCA Math Bonanza Individual Round#10

I11 Let G be the centroid of triangle ABC with $AB = 13, BC = 14, CA = 15$. Calculate the sum of the distances from G to the three sides of the triangle.

Note: The *centroid* of a triangle is the point that lies on each of the three line segments between a vertex and the midpoint of its opposite side.

2019 CCA Math Bonanza Individual Round#11

I12 Let $f(x,y) = x^2\left((x+2y)^2 - y^2 + x - 1\right)$. If $f(a, b+c) = f(b, c+a) = f(c, a+b)$ for distinct numbers a, b, c, what are all possible values of $a+b+c$?

2019 CCA Math Bonanza Individual Round#12

I13 Convex quadrilateral $ABCD$ has $AB = 20$, $BC = CD = 26$, and $\angle ABC = 90^\circ$. Point P is on DA such that $\angle PBA = \angle ADB$. If $PB = 20$, compute the area of $ABCD$.

2019 CCA Math Bonanza Individual Round#13

I14 Call an odd prime p *adjective* if there exists an infinite sequence $a_0, a_1, a_2, \ldots$ of positive integers such that

$$a_0 \equiv 1 + \frac{1}{a_1} \equiv 1 + \frac{1}{1 + \frac{1}{a_2}} \equiv 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{a_3}}} \equiv \ldots \pmod{p}.$$

What is the sum of the first three odd primes that are *not* adjective?

Note: For two common fractions $\frac{a}{b}$ and $\frac{c}{d}$, we say that $\frac{a}{b} \equiv \frac{c}{d} \pmod{p}$ if p divides $ad - bc$ and p does not divide bd.

2019 CCA Math Bonanza Individual Round#14

I15 Before Harry Potter died, he decided to bury his wand in one of eight possible locations (uniformly at random). A squad of Death Eaters decided to go hunting for the wand. They know the eight locations but have poor vision, so even if they're at the correct location they only have a 50% chance of seeing the wand. They also get tired easily, so they can only check three different locations a day. At least they have one thing going for them: they're clever. Assuming they strategize optimally, what is the expected number of days it will take for them to find the wand?

2019 CCA Math Bonanza Individual Round#15

– Team Round

T1 Will has a sock drawer with 2 socks of each color: red, green, blue, white, black (socks of the same color are indistinguishable). He absentmindedly grabs 2 socks out of the drawer. What is the probability that he gets a pair of matching socks?

2019 CCA Math Bonanza Team Round#1

T2 A triangle has side lengths of $x, 75, 100$ where $x < 75$ and altitudes of lengths $y, 28, 60$ where $y < 28$. What is the value of $x + y$?

2019 CCA Math Bonanza Team Round#2

T3 What is the sum of all possible values of $\cos(2\theta)$ if $\cos(2\theta) = 2\cos(\theta)$ for a real number θ?

2019 CCA Math Bonanza Team Round#3

T4 Find the number of ordered tuples (C, A, M, B) of non-negative integers such that

$$C! + C! + A! + M! = B!$$

2019 CCA Math Bonanza Team Round#4

T5 What is the smallest positive integer n such that there exists a choice of signs for which

$$1^2 \pm 2^2 \pm 3^2 \ldots \pm n^2 = 0$$

is true?

2019 CCA Math Bonanza Team Round#5

T6 Compute $\displaystyle\sum_{n=3}^{\infty} \frac{n^2 - 2}{(n^2 - 1)(n^2 - 4)}$.

2019 CCA Math Bonanza Team Round#6

T7 How many ordered triples (a, b, c) of positive integers are there such that at least two of a, b, c are prime and $abc = 11\,(a + b + c)$?

2019 CCA Math Bonanza Team Round#7

T8 fantasticbobob is proctoring a room for the SiSiEyMB with 841 seats arranged in 29 rows and 29 columns. The contestants sit down, take part 1 of the contest, go outside for a break, and come back to take part 2 of the contest. fantasticbobob sits among the contestants during part 1, also goes outside during break, but when he returns, he finds that his seat has been taken. Furthermore, each of the 840 contestants now sit in a chair horizontally or vertically adjacent to their original chair. How many seats could fantasticbobob have started in?

2019 CCA Math Bonanza Team Round#8

T9 Points P, Q, and M lie on a circle ω such that M is the midpoint of minor arc PQ and $MP = MQ = 3$. Point X varies on major arc PQ, MX meets segment PQ at R, the line through R perpendicular to MX meets minor arc PQ at S, MS meets line PQ at T. If $TX = 5$ when MS is minimized, what is the minimum value of MS?

2019 CCA Math Bonanza Team Round#9

T10 Define three sequences a_n, b_n, c_n as $a_0 = b_0 = c_0 = 1$ and

$$\begin{aligned} a_{n+1} &= a_n + 3b_n + 3c_n \\ b_{n+1} &= a_n + b_n + 3c_n \\ c_{n+1} &= a_n + b_n + c_n \end{aligned}$$

for $n \geq 0$. Let A, B, C be the remainders when $a_{13^4}, b_{13^4}, c_{13^4}$ are divided by 13. Find the ordered triple (A, B, C).

2019 CCA Math Bonanza Team Round#10

– Lightning Round

L1.1 How many integers divide either 2018 or 2019? Note: 673 and 1009 are both prime.

2019 CCA Math Bonanza Lightning Round#1.1

L1.2 At Kanye Crest Academy, employees get paid in CCA Math Bananas™. At the end of 2018, Professor Shian Bray was given a 10% pay raise from his salary at the end of 2017. However, inflation caused the worth of a CCA Math Banana™ to decrease by 1%. If Prof. Bray's salary at the end of 2017 was worth one million dollars, how much (in dollars) was Prof. Bray's salary worth at the end of 2018? Assume that the value of the dollar has not changed.

2019 CCA Math Bonanza Lightning Round#1.2

L1.3 Points P and Q are chosen on diagonal AC of square $ABCD$ such that $AB = AP = CQ = 1$. What is the measure of $\angle PBQ$ in degrees?

2019 CCA Math Bonanza Lightning Round#1.3

L1.4 What is the smallest prime number p such that $1 + p + p^2 + \ldots + p^{p-1}$ is *not* prime?

2019 CCA Math Bonanza Lightning Round#1.4

L2.1 Noew is writing a 15-problem mock AIME consisting of four subjects of problems: algebra, geometry, combinatorics, and number theory. The AIME is considered *somewhat evenly distributed* if there is at least one problem of each subject and there are at least six combinatorics problems. Two AIMEs are considered *similar* if they have the same subject distribution (same number of each subject). How many non-similar somewhat evenly distributed mock AIMEs can Noew write?

2019 CCA Math Bonanza Lightning Round#2.1

L2.2 What is the largest positive integer n for which there are no *positive* integers a, b such that $8a + 11b = n$?

2019 CCA Math Bonanza Lightning Round#2.2

L2.3 Compute $\sin^4(7.5^\circ) + \sin^4(82.5^\circ)$.

2019 CCA Math Bonanza Lightning Round#2.3

L2.4 Let $ABCD$ be a parallelogram. Let G, H be the feet of the altitudes from A to CD and BC respectively. If $AD = 15$, $AG = 12$, and $AH = 16$, find the length of AB.

2019 CCA Math Bonanza Lightning Round#2.4

L3.1 Suppose that N is a three digit number divisible by 7 such that upon removing its middle digit, the remaining two digit number is also divisible by 7. What is the minimum possible value of N?

2019 CCA Math Bonanza Lightning Round#3.1

L3.2 What is the area of a triangle with side lengths 17, 25, and 26?

2019 CCA Math Bonanza Lightning Round#3.2

L3.3 64 teams with distinct skill levels participate in a knockout tournament. In each of the 6 rounds, teams are paired into match-ups and compete; the winning team moves on to the next round and the losing team is eliminated. After the second-to-last round, winners compete for first and second and losers compete for third and fourth. Assume that the team with higher skill

level always wins. What is the probability that the first, second, and third place teams have the highest, second highest, and third highest skill levels, respectively?

2019 CCA Math Bonanza Lightning Round#3.3

L3.4 Determine the maximum possible value of

$$\frac{\left(x^2+5x+12\right)\left(x^2+5x-12\right)\left(x^2-5x+12\right)\left(-x^2+5x+12\right)}{x^4}$$

over all non-zero real numbers x.

2019 CCA Math Bonanza Lightning Round#3.4

L4.1 The Garfield Super Winners play 100 games of foosball, in which teams score a non-negative integer number of points and the team with more points after ten minutes wins (if both teams have the same number of points, it is a draw). Suppose that the Garfield Super Winners score an average of 7 points per game but allow an average of 8 points per game. Given that the Garfield Super Winners never won or lost by more than 10, what is the largest possible number of games that they could win?

2019 CCA Math Bonanza Lightning Round#4.1

L4.2 GM Bisain's IQ is so high that he can move around in 10 dimensional space. He starts at the origin and moves in a straight line away from the origin, stopping after 3 units. How many lattice points can he land on? A lattice point is one with all integer coordinates.

2019 CCA Math Bonanza Lightning Round#4.2

L4.3 Let ABC be a triangle with area K. Points A^*, B^*, and C^* are chosen on AB, BC, and CA respectively such that $\triangle A^*B^*C^*$ has area J. Suppose that

$$\frac{AA^*}{AB}=\frac{BB^*}{BC}=\frac{CC^*}{CA}=\frac{J}{K}=x$$

for some $0<x<1$. What is x?

2019 CCA Math Bonanza Lightning Round#4.3

L4.4 If an angle $0^\circ<\theta<30^\circ$ satisfies $\sin\left(90^\circ-\theta\right)\sin\left(60^\circ-\theta\right)\sin\left(30^\circ-\theta\right)=\sin^3\left(\theta\right)$, compute $\sin\left(\theta\right)$.

2019 CCA Math Bonanza Lightning Round#4.4

L5.1 Let $F_1=F_2=1$ and $F_n=F_{n-1}+F_{n-2}$ for any integer $n\geq 3$. For some integer $k>1$, Johnny converts F_k kilometers to miles, then rounds to the nearest integer. Assume that 1 mile is exactly 1.609344 kilometers. Estimate the smallest value of k such that Johnny *does not* get that this is F_{k-1} miles. An estimate of E earns $2^{1-|A-E|}$ points, where A is the actual answer.

2019 CCA Math Bonanza Lightning Round#5.1

L5.2 Suppose that a planet contains $(CCAMATHBONANZA_{71})^{100}$ people (100 in decimal), where in base 71 the digits $A, B, C, \ldots, Z$ represent the decimal numbers $10, 11, 12, \ldots, 35$, respectively. Suppose that one person on this planet is snapping, and each time they snap, at least half of the current population disappears. Estimate the largest number of times that this person can snap without disappearing. An estimate of E earns $2^{1-\frac{1}{200}|A-E|}$ points, where A is the actual answer.

2019 CCA Math Bonanza Lightning Round#5.2

L5.3 For a positive integer n, let $d(n)$ be the number of positive divisors of n (for example $d(39) = 4$). Estimate the average value that $d(n)$ takes on as n ranges from 1 to 2019. An estimate of E earns $2^{1-|A-E|}$ points, where A is the actual answer.

2019 CCA Math Bonanza Lightning Round#5.3

L5.4 Submit an integer between 0 and 100 inclusive as your answer to this problem. Suppose that Q_1 and Q_3 are the medians of the smallest 50% and largest 50% of submissions for this question. Your goal is to have your submission close to $D = Q_3 - Q_1$. If you submit N, your score will be $2 - 2\sqrt{\frac{|N-D|}{\max\{D, 100-D\}}}$.

2019 CCA Math Bonanza Lightning Round#5.4

– Tiebreaker Round

TB1 Compute $1^4 + 2^4 + 3^4 + 4^4 + 5^4 + 6^4$.

2019 CCA Math Bonanza Tiebreaker Round#1

TB2 Isosceles triangle $\triangle ABC$ has $\angle ABC = \angle ACB = 72^\circ$ and $BC = 1$. If the angle bisector of $\angle ABC$ meets AC at D, what is the positive difference between the perimeters of $\triangle ABD$ and $\triangle BCD$?

2019 CCA Math Bonanza Tiebreaker Round#2

TB3 For $i = 1, 2, \ldots, 7$, Zadam Heng chooses a positive integer m_i at random such that each positive integer k is chosen with probability $\frac{2^i-1}{2^{ik}}$. If $m_1 + 2m_2 + \ldots + 7m_7 \neq 35$, Zadam keeps rechoosing the m_i until this equality holds. Given that he eventually stops, what is the probability that $m_4 = 1$ when Zadam stops?

2019 CCA Math Bonanza Tiebreaker Round#3

TB4 The number $28!$ (28 in decimal) has base 30 representation

$$28! = Q6T32S??OCLQJ6000000_{30}$$

where the seventh and eighth digits are missing. What are the missing digits? In base 30, we have that the digits $A = 10$, $B = 11$, $C = 12$, $D = 13$, $E = 14$, $F = 15$, $G = 16$, $H = 17$, $I = 18$, $J = 19$, $K = 20$, $L = 21$, $M = 22$, $N = 23$, $O = 24$, $P = 25$, $Q = 26$, $R = 27$, $S = 28$, $T = 29$.

2019 CCA Math Bonanza Tiebreaker Round#4