**Japan MO Finals 2019**

www.artofproblemsolving.com/community/c831682

by Kunihiko_Chikaya

**1** Find all triples $(a,\ b,\ c)$ of positive integers such that

$$a^2+b+3=(b^2-c^2)^2.$$

---

**2** Let $n\geq 3$ be an odd number. We will play a game using a $n$ by $n$ grid. The game is comprised of $n^2$ turns, in every turn, we will perform the following operation sequentially.

• We will choose a square with an unwritten integer, and write down an integer among 1 through $n^2$. We can write down any integer only at once through the game.

• For each row, colum including the square, if the sum of integers is a multiple of $n$, then we will get 1 point (both of each sum is a multiple of $n$, we will get 2 points).

Determine the maximum possible value of the points as the total sum that we can obtain by the end of the game.

---

**3** Find all functions $f:\mathbb{R}^+\rightarrow\mathbb{R}^+$ such that

$$f\left(\frac{f(y)}{f(x)}+1\right)=f\left(x+\frac{y}{x}+1\right)-f(x)$$

for all $x,\ y\in\mathbb{R}^+$.

---

**4** Let $ABC$ be a triangle with its inceter $I$, incircle $w$, and let $M$ be a midpoint of the side $BC$. A line through the point $A$ perpendicular to the line $BC$ and a line through the point $M$ perpendicular to the line $AI$ meet at $K$. Show that a circle with line segment $AK$ as the diameter touches $w$.

---

**5** Let $S$ be a set which is comprised of positive integers. We call $S$ a $beautiful\ number$ when the element belonging to $S$ of which any two distinct elements $x,\ y,\ z$, at least of them will be a divisor of $x+y+z$. Show that there exists an integer $N$ satisfying the following condition, and also determine the smallest $N$ as such :

For any set S of $beautiful\ number$, there exists, $n_s\geq 2$ being an integer, the number of the element belonging to $S$ which is not a multiple of $n_s$, is less than or equal to $N$.

---