China Second Round Olympiad 2017

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–	Test 1
2	Let x,y are real numbers such that $x^2+2cosy=1$. Find the ranges of $x-cosy$.
10	Let $x_1,x_2,x_3\geq 0$ and $x_1+x_2+x_3=1$. Find the minimum value and the maximum value of $(x_1+3x_2+5x_3)\left(x_1+\frac{x_2}{3}+\frac{x_3}{5}\right)$.
–	Test 2
1	Given an isocleos triangle ABC with equal sides $AB=AC$ and incenter I.Let $\Gamma_1$be the circle centered at A with radius AB,Γ_2 be the circle centered at I with radius BI.A circle Γ_3 passing through B,I intersects Γ_1,Γ_2 again at P,Q (different from B) respectively.Let R be the intersection of PI and BQ.Show that $BR\perp CR$.
2	Given a sequence $\{a_n\}$: $a_1=1,a_{n+1}=\begin{cases} a_n+n, & a_n\leq n, \\ a_n-n, & a_n>n, \end{cases} \quad n=1,2,\cdots$. Find the number of positive integers r satisfying $a_r<r\leq 3^{2017}$.
3	Each square of a 33×33 square grid is colored in one of the three colors: red, yellow or blue, such that the numbers of squares in each color are the same. If two squares sharing a common edge are in different colors, call that common edge a separating edge. Find the minimal number of separating edges in the grid.
4	Let m,n be integers greater than 1,$m\geq n$,$a_1,a_2,\dots,a_n$ are n distinct numbers not exceed m,which are relatively primitive.Show that for any real x,there exists i for which $\|\|a_ix\|\|\geq\frac{2}{m(m+1)}\|\|x\|\|$,where $\|\|x\|\|$ denotes the distance between x and the nearest integer to x .

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