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1 Find all pairs of positive integers (n,k) so that $(n+1)^k - 1 = n!$.

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2 in the acute triangle $\triangle ABC$.
M is a point in the interior of the segment AC and N is a point on the extension of segment AC such that MN=AC.
let D,E be the feet of perpendiculars from M,N onto lines BC,AB respectively
prove that the orthocentre of $\triangle ABC$ lies on circumcircle of $\triangle BED$

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3 let n,m be positive integers st $m > n \geq 5$ with m depending on n.
consider the sequence $a_1, a_2, ...a_m$ where $a_i = i$ for $i = 1, ..., n$ $a_{n+j} = a_{3j} + a_{3j-1} + a_{3j-2}$ for $j = 1, .., m-n$
with $m - 3(m-n)$ =1 or 2, ie $a_m = a_{m-k} + a_{m-k-1} + a_{m-k-2}$ where k=1 or 2
(Thus if $n = 5$, the sequence is 1,2,3,4,5,6,15
and if $n = 8$, the sequence is 1,2,3,4,5,6,7,8,6,15,21)
Find $S = a_1 + ... + a_m$ if (i) $n = 2007$ (ii) $n = 2008$

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4 let $0 < a, b < \pi/2$. Show that $\frac{5}{cos^2(a)} + \frac{5}{sin^2(a)sin^2(b)cos^2(b)} \geq 27cos(a) + 36sin(a)$

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5 consider a 2008×2008 chess board. let M be the smallest no of rectangles that can be drawn on the chess board so that sides of every cell of the board is contained in the sides of one of the rectangles. find the value of M. (eg for 2×3 chessboard, the value of M is 3.)
