Federal Competition For Advanced Students, Part 1 2017

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by matinyousefi

1 Determine all polynomials $P(x) \in \mathbb{R}[x]$ satisfying the following two conditions :
(a) $P(2017) = 2016$ and
(b) $(P(x)+1)^2 = P(x^2+1)$ for all real numbers x.

proposed by Walther Janous

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2 Let $ABCDE$ be a regular pentagon with center M. A point P (different from M) is chosen on the line
segment MD. The circumcircle of ABP intersects the line segment AE in A and Q and
the line through P perpendicular to CD in P and R.
Prove that AR and QR have same length.

proposed by Stephan Wagner

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3 Anna and Berta play a game in which they take turns in removing marbles from a table. Anna takes the first turn. At the beginning of a turn there are n 1 marbles on the table, then the player whose turn is removes k marbles, where k 1 either is an even number with $k \le \frac{n}{2}$ or an odd number with $\frac{n}{2} \le k \le n$. A player wins the game if she removes the last marble from the table.
Determine the smallest number $N \ge 100000$ which Berta has wining strategy.

proposed by Gerhard Woeginger

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4 Find all pairs (a, b) of non-negative integers such that:

$$2017^a = b^6 - 32b + 1$$

proposed by Walther Janous
