International Zhautykov Olympiad 2019

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1 Prove that there exist at least $100!$ ways to write $100!$ as sum of elements of set $1!, 2!, 3!...99!$ (each number in sum can be two or more times)

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2 Find the biggest real number C, such that for every different positive real numbers $a_1, a_2...a_{2019}$ that satisfy inequality : $\frac{a_1}{|a_2-a_3|} + \frac{a_2}{|a_3-a_4|} + ... + \frac{a_{2019}}{|a_1-a_2|} > C$

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3 Triangle ABC is given. The median CM intersects the circumference of ABC in N. P and Q are chosen on the rays CA and CB respectively, such that PM is parallel to BN and QM is parallel to AN. Points X and Y are chosen on the segments PM and QM respectively, such that both PY and QX touch the circumference of ABC. Let Z be intersection of PY and QX. Prove that, the quadrilateral $MXZY$ is circumscribed.

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4 Triangle ABC with $AC = BC$ given and point D is chosen on the side AC. $S1$ is a circle that touches AD and extensions of AB and BD with radius R and center O_1. $S2$ is a circle that touches CD and extensions of BC and BD with radius $2R$ and center O_2. Let F be intersection of the extension of AB and tangent at O_2 to circumference of BO_1O_2. Prove that $FO_1 = O_1O_2$.

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5 Natural number $n > 1$ is given. Let I be a set of integers that are relatively prime to n. Define the function $f : I => N$. We call a function $k - periodic$ if for any a, b , $f(a) = f(b)$ whenever $k|a - b$. We know that f is $n - periodic$. Prove that minimal period of f divides all other periods. Example: if $n = 6$ and $f(1) = f(5)$ then minimal period is 1, if $f(1)$ is not equal to $f(5)$ then minimal period is 3.

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6 We define two types of operation on polynomial of third degree:
a) switch places of the coefficients of polynomial(including zero coefficients), ex: $x^3 + x^2 + 3x - 2$ =¿ $-2x^3 + 3x^2 + x + 1$
b) replace the polynomial $P(x)$ with $P(x + 1)$
If limitless amount of operations is allowed,
is it possible from $x^3 - 2$ to get $x^3 - 3x^2 + 3x - 3$?
