Kazakhstan National Olympiad 2019

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– [b] Day 1

1 Prove for any positives a, b, c the inequality

$$\sqrt[3]{\frac{a}{b}}+\sqrt[5]{\frac{b}{c}}+\sqrt[7]{\frac{c}{a}}>\frac{5}{2}$$

2 The set consists of a finite number of points on the plane. The distance between any two points from is at least $\sqrt{2}$. It is known that a regular triangle with side lenght 3 cut out of paper can cover all points of . What is the greatest number of points that can consist of?

3 Let p be a prime number of the form $4k+1$ and $\frac{m}{n}$ is an irreducible fraction such that

$$\sum_{a=2}^{p-2}\frac{1}{a^{(p-1)/2}+a^{(p+1)/2}}=\frac{m}{n}.$$

Prove that $p|m+n$.

(Fixed, thanks Pavel)

– [b] Day 2

4 Find all positive integers $n, k, a_1, a_2, ..., a_k$ so that $n^{k+1}+1$ is divisible by $(na_1+1)(na_2+1)...(na_k+1)$

5 Given a checkered rectangle of size n m. Is it always possible to mark 3 or 4 nodes of a rectangle so that at least one of the marked nodes lay on each straight line containing the side of the rectangle, and the non-self-intersecting polygon with vertices at these nodes has an area equal to

$$\frac{1}{2}\min\left(\text{gcd}(n,m),\frac{n+m}{\text{gcd}(n,m)}\right)$$

?

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6 The tangent line l to the circumcircle of an acute triangle ABC intersects the lines AB, BC, and CA at points C', A' and B', respectively. Let H be the orthocenter of a triangle ABC. On the straight lines A'H, BH and C'H, respectively, points A_1, B_1 and C_1 (other than H) are marked such that $AH = AA_1, BH = BB_1$ and $CH = CC_1$. Prove that the circumcircles of triangles ABC and $A_1B_1C_1$ are tangent.

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