Azerbaijan IMO TST 2015

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– First Day

1 We say that A=$a_1, a_2, a_3 \cdots a_n$ consisting $n > 2$ distinct positive integers is $good$ if for every $i = 1, 2, 3 \cdots n$ the number ${a_i}^{2015}$ is divisible by the product of all numbers in A except a_i. Find all integers $n > 2$ such that exists a $good$ set consisting of n positive integers.

2 Find all functions $f : [0, 1] \to \mathbb{R}$ such that the inequality

$$(x-y)^2 \leq |f(x) - f(y)| \leq |x-y|$$

is satisfied for all $x, y \in [0, 1]$

3 Consider a trapezoid $ABCD$ with $BC||AD$ and $BC < AD$. Let the lines AB and CD meet at X. Let ω_1 be the incircle of the triangle XBC, and let ω_2 be the excircle of the triangle XAD which is tangent to the segment AD . Denote by a and d the lines tangent to ω_1 , distinct from AB and CD, and passing through A and D, respectively. Denote by b and c the lines tangent to ω_2 , distinct from AB and CD, passing through B and C respectively. Assume that the lines a, b, c and d are distinct. Prove that they form a parallelogram.

– Second Day

1 Let ω be the circumcircle of an acute-angled triangle ABC. The lines tangent to ω at the points A and B meet at K. The line passing through K and parallel to BC intersects the side AC at S. Prove that $BS = CS$

2 Alex and Bob play a game 2015 x 2015 checkered board by the following rules.Initially the board is empty: the players move in turn, Alex moves first. By a move, a player puts either red or blue token into any unoccopied square. If after a player's move there appears a row of three consecutive tokens of the same color(this row may be vertical,horizontal, or dioganal), then this player wins. If all the cells are occupied by tokens, but no such row appears, then a draw is declared.Determine whether Alex, Bob, or none of them has winning strategy.

3 Let n and k be two positive integers such that $n > k$. Prove that the equation $x^n + y^n = z^k$ has a solution in positive integers if and only if the equation $x^n + y^n = z^{n-k}$ has a solution in positive integers.