

Austria Beginners' Competition 2015

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by RockmanEX3

- 1 Let a, b, c be integers with $a^3 + b^3 + c^3$ divisible by 18. Prove that abc is divisible by 6.

(Karl Czakler)

- 2 Let x, y be positive real numbers with $xy = 4$.

Prove that

$$\frac{1}{x+3} + \frac{1}{y+3} \leq \frac{2}{5}$$

For which x and y does equality hold?

(Walther Janous)

- 3 Anton chooses as starting number an integer $n \geq 0$ which is not a square. Berta adds to this number its successor $n + 1$. If this sum is a perfect square, she has won. Otherwise, Anton adds to this sum, the subsequent number $n + 2$. If this sum is a perfect square, he has won. Otherwise, it is again Berta's turn and she adds the subsequent number $n + 3$, and so on.

Prove that there are infinitely many starting numbers, leading to Anton's win.

(Richard Henner)

- 4 Let k_1 and k_2 be internally tangent circles with common point X . Let P be a point lying neither on one of the two circles nor on the line through the two centers. Let N_1 be the point on k_1 closest to P and F_1 the point on k_1 that is farthest from P . Analogously, let N_2 be the point on k_2 closest to P and F_2 the point on k_2 that is farthest from P .

Prove that $\angle N_1 X N_2 = \angle F_1 X F_2$.

(Robert Geretschlger)
