Tajikistan Team Selection Test 2014

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by umedkarimov

1 Given the polynomial $p(x) = x^2 + x - 70$, do there exist integers $0 < m < n$, so that $p(m)$ is divisible by n and $p(m+1)$ is divisible by $n+1$?

Proposed by Nairy Sedrakyan

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2 Let Mbe an interior point of triangle ABC. Let the line AM intersect the circumcircle of the triangle MBC for the second time at point D, the line BM intersect the circumcircle of the triangle MCA for the second time at point E, and the line CM intersect the circumcircle of the triangle MAB for the second time at point F. Prove that $\frac{AD}{MD} + \frac{BE}{ME} + \frac{CF}{MF} \geq \frac{9}{2}$.

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3 Let a, b, c be side length of a triangle. Prove the inequality

$$\sqrt{a^2 + ab + b^2} + \sqrt{b^2 + bc + c^2} + \sqrt{c^2 + ca + a^2} \leq \sqrt{5a^2 + 5b^2 + 5c^2 + 4ab + 4bc + 4ca}.$$

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4 In a convex hexagon $ABCDEF$ the diagonals AD, BE, CF intersect at a point M. It is known that the triangles $ABM, BCM, CDM, DEM, EFM, FAM$ are acute. It is also known that the quadrilaterals $ABDE, BCEF, CDFA$ have the same area. Prove that the circumcenters of triangles $ABM, BCM, CDM, DEM, EFM, FAM$ are concyclic.

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5 There are 12 delegates in a mathematical conference. It is known that every two delegates share a common friend. Prove that there is a delegate who has at least five friends in that conference.

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