Austria Beginners' Competition 2016

www.artofproblemsolving.com/community/c854169

by RockmanEX3

1 Determine all nonnegative integers n having two distinct positive divisors with the same distance from $\frac{n}{3}$.

(Richard Henner)

---

2 Prove that all real numbers $x \neq -1$, $y \neq -1$ with $xy = 1$ satisfy the following inequality:

$$\left(\frac{2+x}{1+x}\right)^2 + \left(\frac{2+y}{1+y}\right)^2 \geq \frac{9}{2}$$

(Karl Czakler)

---

3 We consider the following gure:

[See attachment]

We are looking for labellings of the nine elds with the numbers 1, 2, ..., 9. Each of these numbers has to be used exactly once. Moreover, the six sums of three resp. four numbers along the drawn lines have to be be equal. Give one such labelling. Show that all such labellings have the same number in the top eld. How many such labellings do there exist? (Two labellings are considered dierent, if they disagree in at least one eld.)

(Walther Janous)

---

4 Let $ABCDE$ be a convex pentagon with ve equal sides and right angles at C and D. Let P denote the intersection point of the diagonals AC and BD. Prove that the segments PA and PD have the same length.

(Gottfried Perz)
