Federal Competition For Advanced Students, Part 1, 2016

www.artofproblemsolving.com/community/c854175

by RockmanEX3

1 Determine the largest constant C such that

$$(x_1 + x_2 + \cdots + x_6)^2 \geq C \cdot (x_1(x_2 + x_3) + x_2(x_3 + x_4) + \cdots + x_6(x_1 + x_2))$$

holds for all real numbers $x_1, x_2, \cdots, x_6$.

For this C, determine all $x_1, x_2, \cdots x_6$ such that equality holds.

(Walther Janous)

---

2 We are given an acute triangle ABC with $AB > AC$ and orthocenter H. The point E lies symmetric to C with respect to the altitude AH. Let F be the intersection of the lines EH and AC. Prove that the circumcenter of the triangle AEF lies on the line AB.

(Karl Czakler)

---

3 Consider 2016 points arranged on a circle. We are allowed to jump ahead by 2 or 3 points in clockwise direction.

What is the minimum number of jumps required to visit all points and return to the starting point?

(Gerd Baron)

---

4 Determine all composite positive integers n with the following property: If $1 = d_1 < d_2 < \cdots < d_k = n$ are all the positive divisors of n, then

$$(d_2 - d_1) : (d_3 - d_2) : \cdots : (d_k - d_{k-1}) = 1 : 2 : \cdots : (k-1)$$

(Walther Janous)
