

**Mediterranean Mathematics Olympiad 2018**

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- 1 Let  $a_1, a_2, \dots, a_n$  be more than one real numbers, such that  $0 \leq a_i \leq \frac{\pi}{2}$ . Prove that

$$\left( \frac{1}{n} \sum_{i=1}^n \frac{1}{1 + \sin a_i} \right) \left( 1 + \prod_{i=1}^n (\sin a_i)^{\frac{1}{n}} \right) \leq 1.$$

- 2 Let  $ABC$  be acute triangle. Let  $E$  and  $F$  be points on  $BC$ , such that angles  $BAE$  and  $FAC$  are equal. Lines  $AE$  and  $AF$  intersect circumcircle of  $ABC$  at points  $M$  and  $N$ . On rays  $AB$  and  $AC$  we have points  $P$  and  $R$ , such that angle  $PEA$  is equal to angle  $B$  and angle  $AER$  is equal to angle  $C$ . Let  $L$  be intersection of  $AE$  and  $PR$  and  $D$  be intersection of  $BC$  and  $LN$ . Prove that

$$\frac{1}{|MN|} + \frac{1}{|EF|} = \frac{1}{|ED|}.$$

- 3 An integer  $a \geq 1$  is called *Aegean*, if none of the numbers  $a^{n+2} + 3a^n + 1$  with  $n \geq 1$  is prime. Prove that there are at least 500 Aegean integers in the set  $\{1, 2, \dots, 2018\}$ .

(Proposed by Gerhard Woeginger, Austria)

- 4 Determine the largest integer  $N$ , for which there exists a  $6 \times N$  table  $T$  that has the following properties: \* Every column contains the numbers  $1, 2, \dots, 6$  in some ordering. \* For any two columns  $i \neq j$ , there exists a row  $r$  such that  $T(r, i) = T(r, j)$ . \* For any two columns  $i \neq j$ , there exists a row  $s$  such that  $T(s, i) \neq T(s, j)$ .

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